F- Common Core Math II Name KEY - Revelle

UNIT 7A: Graphing Advanced Functions & Modeling		HOMEWORK
2/24/14 Monday	Linear and Absolute Value Transformations Pages 1-8	
2/25/14 Tuesday	Linear and Absolute Value Transformations Pages 9-13	
2/26/14 Wednesday	Inverse Variation Pages 14-18	
2/27/14 Thursday	Inverse Variation Pages 19-23	
2/28/14 Friday	Radicals Pages 24-27	
3/3/14 Monday	Radicals Pages 28-31	
3/4/14 Tuesday	[Packet 7B] Quadratic & Exponential Review **QUIZ** 32-35	
3/5/14 Wednesday	Power Functions Pages 36-41	
3/6/14 Thursday	Piecewise Functions Pages 42-47	
3/7/14 Friday	Piecewise Functions Pages 48-53	
3/10/14 Monday	Piecewise Functions Pages 54-55	
3/11/14 Tuesday	Step Functions Pages 56-61	
3/12/14 Wednesday	Review Pages 62-70	
3/13/14 Thursday	**Test**	

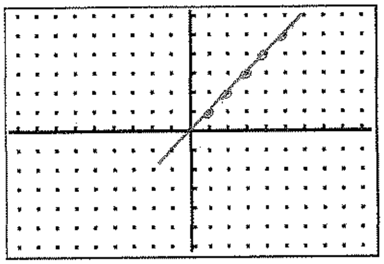
and

given

6.10 Warm Up: Graphing Absolute Value Functions

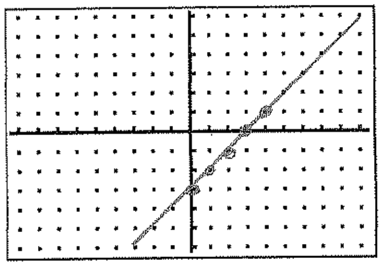
A. (Review): Graph and label intercepts (using ordered pairs) on the graph.

1. $y = x$



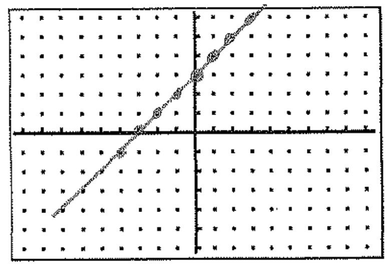
x-intercept: (0,0)

2. $f(x)=x-3$



x-intercept: (3,0)

3. $g(x)=x+3$



x-intercept: (-3,0)

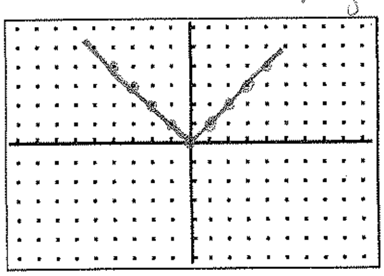
B. Explain your knowledge about **absolute value** using words.

The distance a value is from 0.

The graph is a V shape and made of two rays.

C. Using past knowledge to create new knowledge, try graphing the following function:

4. $y=|x|$ ↗ $y=x$ ↘ $y=-x$



Explain your reasoning for the graph you created.

vertex (0,0)

opens up

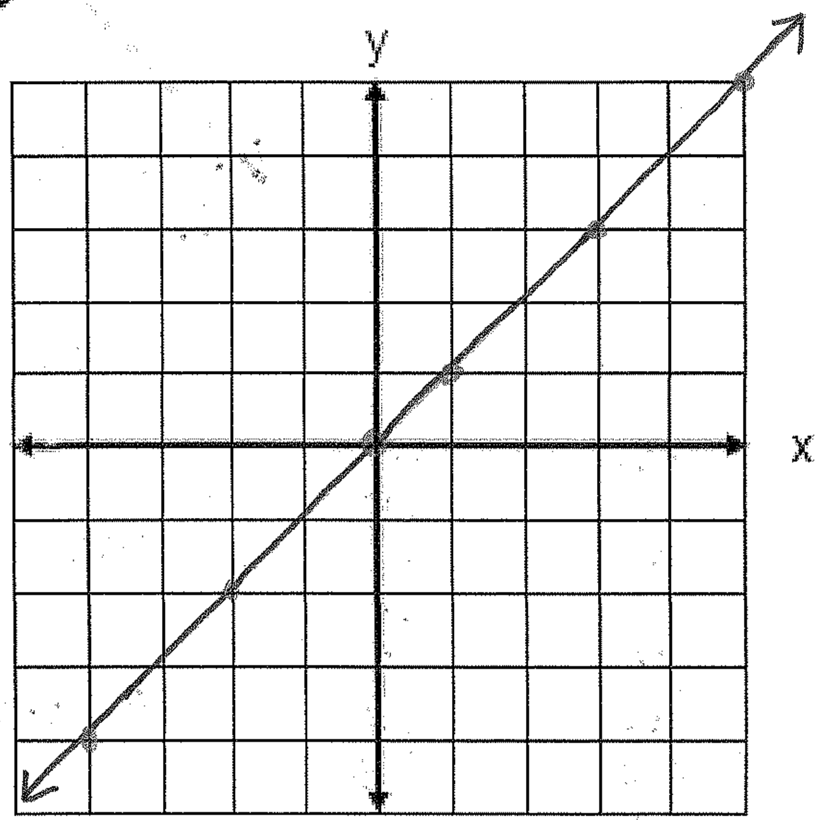
Slope = 1 and -1

Please justify this method (using another method).

x	|x|	y
-3	|-3|	3
-2	|-2|	2
-1	|-1|	1
0	|0|	0
1	|1|	1
2	|2|	2
3	|3|	3

Linear Function
f(x) = x

Plot the points and sketch the graph below.



Complete the table of values.	
x	f (x)
-4	-4
-2	-2
0	0
1	1
3	3
5	5

Why is this called a linear function?
It forms a line.

What is the x-intercept?
(0,0)

What is the y-intercept?
(0,0)

What is the slope?
m=1

Is this graph increasing or decreasing? How do you know? As x increases, y increases.

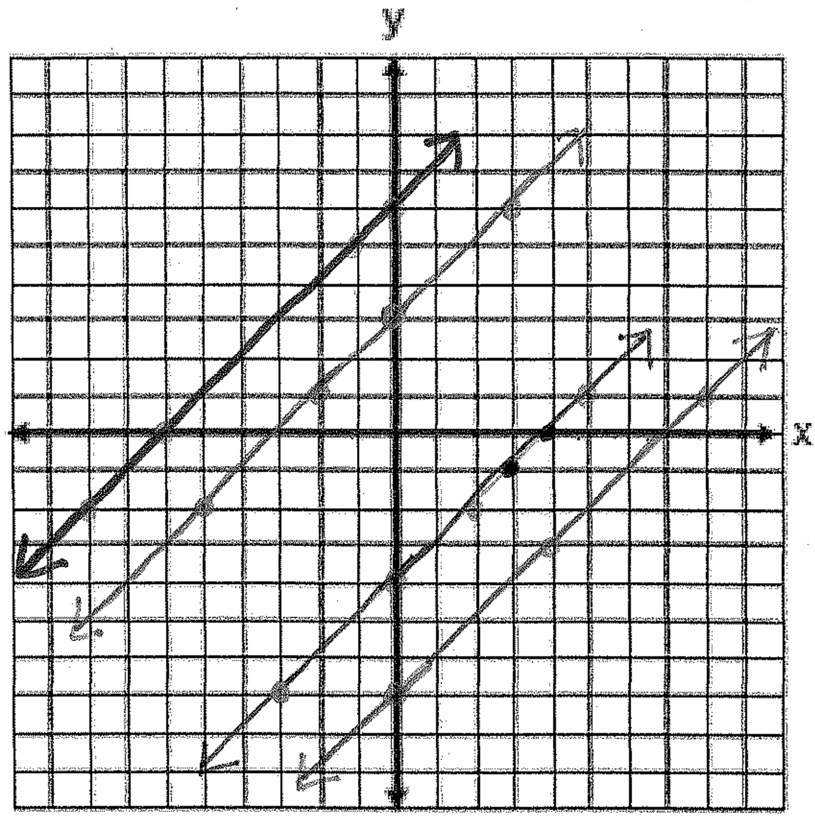
What is the domain and range of this graph? How do you know? all reals $\{x | x = \mathbb{R}\}$

$\{x \in \mathbb{R}\}$

* translations

Families of Linear Functions $f(x) = x + b$

Complete the table on each of the following and draw each in a different color on the graph to the right.



$f(x) = x + 3$	
x	f(x)
-5	-2
-2	1
0	3
3	6
7	10
x-int =	(3,0)
y-int=	(3,0)

$f(x) = x - 4$	
x	f(x)
-6	-10
-3	-7
0	-4
2	-2
5	1
x-int =	(4,0)
y-int=	(0,-4)

$f(x) = x - 7$	
x	f(x)
-2	-9
-1	-8
0	-7
4	-3
8	1
x-int =	(7,0)
y-int=	(0,-7)

$f(x) = x + 6$	
x	f(x)
-8	-2
-6	0
-1	5
2	8
3	9
x-int =	(-6,0)
y-int=	(0,6)

How are the lines above alike?

Same slope, increasing

How are they different?

different y-intercepts and x-intercepts

Write the equation of a line in this family with a y –intercept of –2.

$y=x-2$

Write the equation of a line in this family with a y –intercept of +5.

$y=x+5$

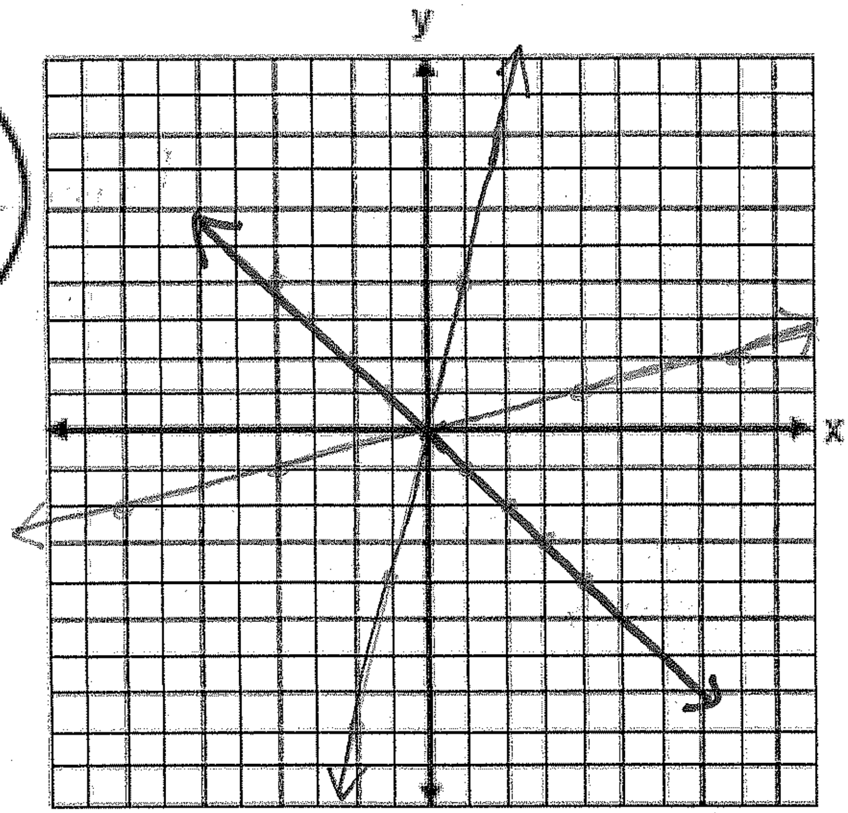
Write the equation of a line in this family with a y –intercept of –10.

$y=x-10$

What is the domain and range of this graph?
How do you know? $\{x \in \mathbb{R}\}$

all reals

Linear Functions in the Form of f(x) = ax



Graph each of the following functions in different colors on the graph at the right.

$f(x) = -x$

$f(x) = \frac{1}{4}x$

$f(x) = 4x$

How are the graphs alike?

They all have the same x + y intercept. They intersect at the origin.

How are the graphs different?

Two increase + one decreases. They have different slopes.

What does the coefficient of x do to the linear function f(x)=x?

It affects the slope (rise or fall).

How would the graph of f(x) = 5x compare to the graph of f(x)=x?

It increases faster

How would the graph of f(x) = -3x compare to the graph of f(x)=x?

It decreases faster.

How would the graph of f(x) = .2x compare to the graph of f(x)=x?

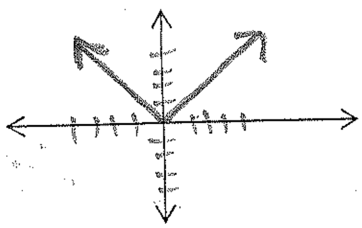
It increases slower

What is the domain and range of this graph? How do you know?

all reals

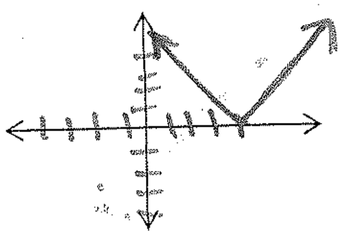
$\{x \in \mathbb{R}\}$

1) **Enter $y = |x|$ into Y1 in your TI-84+ .**
Sketch the graph below.



Parent Function
$y = |x|$

Now sketch the graph of $f(x) = |x - 4|$.



It translated right 4.

The graph of $f(x) = |x - 4|$ is a transformation of $f(x) = |x|$. Describe the transformation:

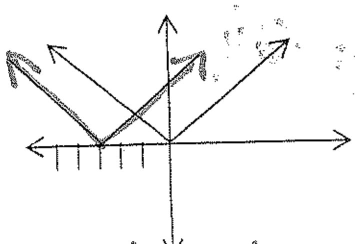
The graph went 4 **units to the** right

Vertex: (h, k)

Find n equation for Y2 that will translate the graph of $f(x) = |x|$ 4 units to the left.

$f(x) =$ $|x+4|$

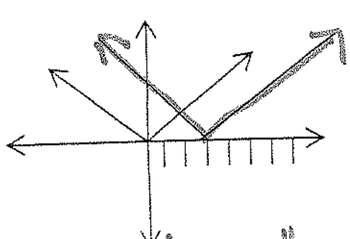
Write a function for Y2 to create each of the transformations of $y = |x|$ below:



left 3

$f(x) = |x+h|$ moves it h units left

$f(x) =$ $|x+3|$



right 3

$f(x) = |x-h|$ moves it h units right

ruff day?

$f(x) =$ $|x-3|$

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U2 L6 LAB: Absolute Value Transformations

Name ____________________ Date ________ Period ____

2) The vertex of the absolute value graph is the critical point of the graph.

critical
turning
minimum/maximum

What is the vertex of $y = |x|$? (0,0)

What is the vertex of $y = |x + 3|$? (-3,0)

How do these two points help you to verify the direction of the transformation? H

shows how the vertex translated to the left 3.

3) Graph $y = |x| + 3$ in Y2. How have you transformed the graph of $y = |x| + 3$?

The graph went 3 **units** up

Name the coordinates of the vertex of $y = |x|$ **and** $y = |x| + 3$

(0,0) (0,3)

Vertex: (h,k)

How do these points help verify the transformation?

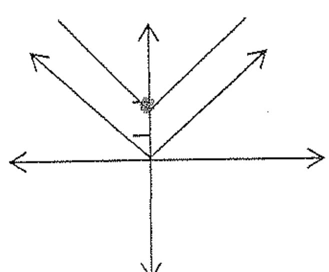
It shows the vertex translated up 3.

$y = |x| + k$ moves up k units

Next, you will transform the graph of $y = |x|$ by making changes to y.

Write a function for Y2 to create each of the graphs below:

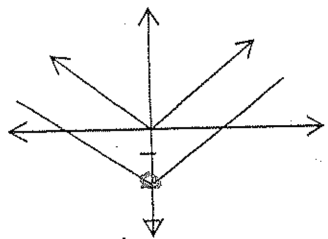
$y = |x| - k$ moves down k units



$f(x) =$ |x| + 2

U2 L6 LAB: Absolute Value Transformations

Name ______________________ Date __________ Period _____



$f(x) =$ $|x| - 2$

Find a translation that will move the graph **up 5 units:**

$f(x) =$ $|x| + 5$

Find a translation that will move the graph **down 1 unit and right 3 units:**

$f(x) =$ $|x - 3| - 1$

Find a translation that will move the graph **up 3 unit and left 2 units:**

$f(x) =$ $|x + 2| + 3$

Summarize what you have discovered about translating the absolute value graph vertically and horizontally:

1) to go up add at the end

2) to go down subtract at the end

3) to go left add inside absolute value

4) to go right subtract inside absolute value

4) Write your own transformation equation and tell how the graph of $y = |x|$ will change:

$f(x) =$ $|x + 3| - 7$

The transformation graph will be

left 3 + down 7

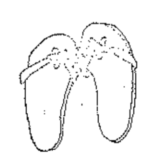
Add on:

$y = 2|x|$

$y = -2|x|$

$y = \frac{1}{2}|x|$

$y = -\frac{1}{2}|x + 4| - 3$



relax

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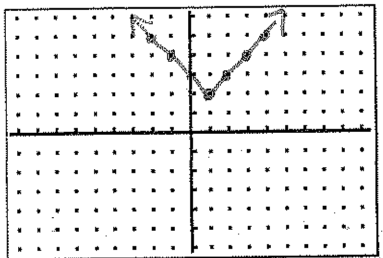
6.10 Homework: Graphing Absolute Value Functions

Graph each of the following functions using a table of values. Then, for each of the functions, state the domain, range, intervals of increasing and decreasing, ~~end behavior~~, minimum or maximum value, vertex, x-intercept(s), and y-intercept.

1. $y = |x-1| + 2$

vertex: (1,2)



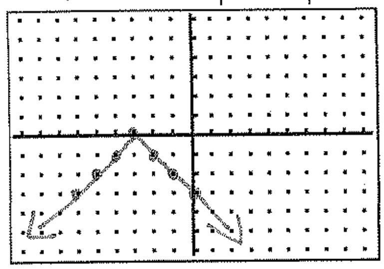
slope = 1 & -1

opens up

domain		max/min	
range		vertex	
increasing		x int	
decreasing		y int	

2. $f(x) = -|x+3|$

vertex: (-3, 0)

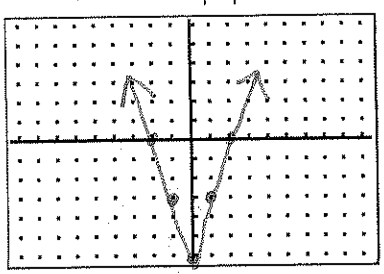


slope = -1 & 1

opens down

3. $g(x) = 3|x| - 6$

vertex: (0, -6)

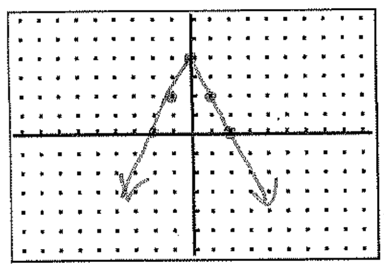


slope = 3 & -3

opens up

4. $y = -2|x| + 4$

vertex: (0,4)



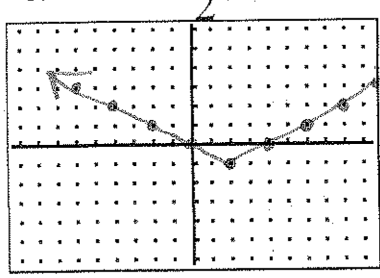
reflects / upside down

slope = -2 & 2

opens down

5. $f(x) = \frac{1}{2}|x-2| - 1$

vertex (2, -1)

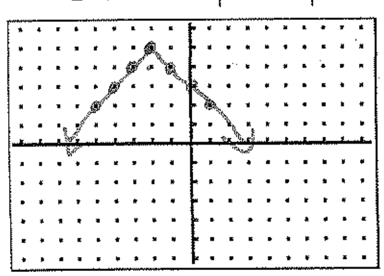


slope = 1/2 & -1/2

opens up

6. $g(x) = -|x+2| + 5$

vertex (-2, 5)



slope = -1 & 1

opens down

domain		max/min	
range		vertex	
increasing		x int	
decreasing		y int	

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Absolute Value Functions

Vocabulary

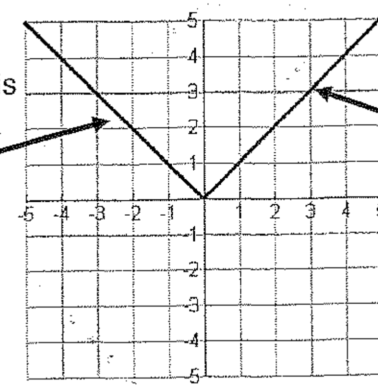
- The function f(x) = |x| is an absolute value (piecewise)

$$|x| = \begin{cases} -x, \ if\ x < 0 \\ 0, \ if\ x = 0 \\ x, \ if\ x > 0 \end{cases}$$

The graph of this piecewise function consists of 2 rays, is V-shaped, and opens up.

To the left of x=0 the line is decreasing

y=-x



To the right of x = 0 the line is increasing

y= x

Notice that the graph is symmetric over the y-axis because for every point (x,y) on the graph, the point (-x,y) is also on it.

Vocabulary

- The highest or lowest point on the graph of an absolute value function is called the vertex.
- An axis of symmetry of the graph of a function is a vertical line that divides the graph into mirror images.
  - **An absolute value graph has one** axis of symmetry **that passes through the** vertex.

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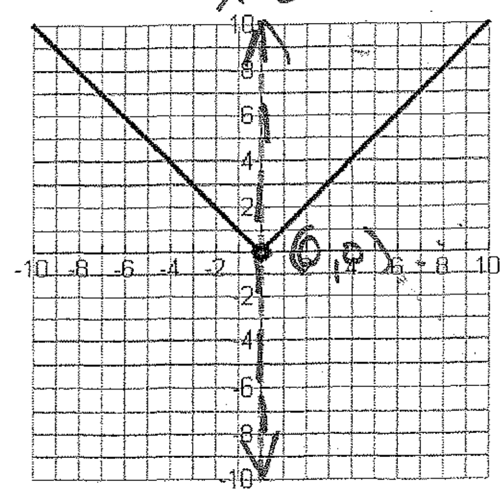
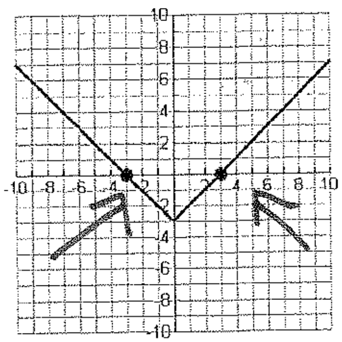
X=0



- **Absolute Value Function** y=|x|
- **Vertex** (0,0)
- **Axis of Symmetry** X=0

Vocabulary

- The <u>Zeros</u> of a function f(x) are the values of x that make the value of f(x) zero.



$f(x) = |x| - 3$

- On this graph where <u>x=3 + x=-3</u> are where the function would equal 0.

Vocabulary

- A <u>transformation</u> changes a graph's size, shape, position, or orientation.
- A <u>translation</u> is a transformation that shifts a graph horizontally and/or vertically, but does not change its size, shape, or orientation.
- A <u>reflection</u> is when a graph is flipped over a line. A graph flips vertically when $-1 \cdot f(x)$ and it flips horizontally when $f(-1x)$.
- A <u>dilation</u> changes the size of a graph by stretching or compressing it. This happens when you multiply the function by a number.

Transformations

$$y = -a\,|x - h| + k$$

reflect x-axis

horizontal compression

a>1 narrows

0<a<1 widens

horizontal stretch

horizontal

vertical

Remember that (h, k) is your <u>Vertex</u>

Example 1:

- Identify the transformations:

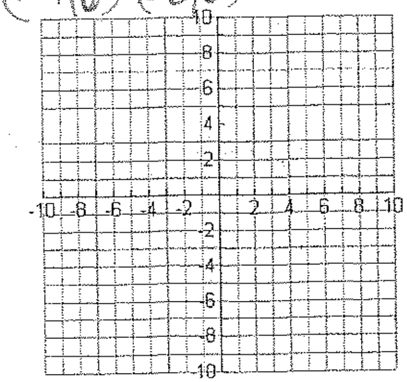
1. $y = 3\,|x + 2| - 3$ narrows, left 2, down 1
2. $y = |x - 1| + 2$ right 1, up 2
3. $y = 2\,|x + 3| - 1$ narrows, left 3, down 1
4. $y = -1/3|x - 2| + 1$ widens, reflects x axis, right 2, up 1

Example 2:

- Graph y = -2 |x + 3| + 2.
  - What is your vertex? (-3, 2)
  - What are the intercepts? (-4, 0) (-2, 0)
  - What are the zeros?

x = -4

x = -2



Slope = -2 + 2

opens down

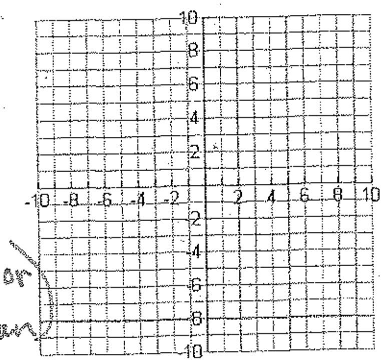
narrows by factor of 2

or horizontal compression of 2

You Try:

- Graph y = -1/2 |x – 1| - 2
  - Compare the graph with the graph of y = |x| (what are the transformations)

vertex (1, -2)



right 1

down 2

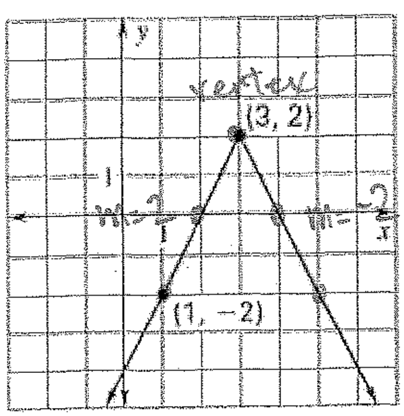
reflects x axis (upside down or opens down)

widens by factor of 1/2

or horizontal stretch of 1/2

Example 3:

- Write a function for the graph shown.



y = □ | x − □ | + □

y = -2 | x - 3 | + 2

opens down: −

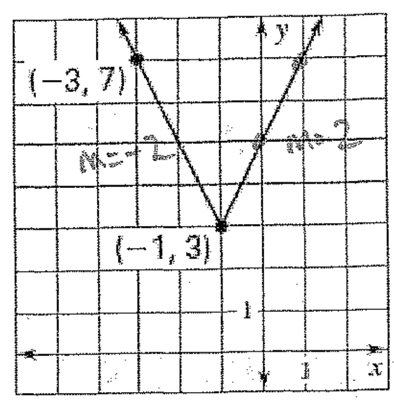
slope : 2 + -2

You Try:

opens up

slope = 2 + -2

- Write a function for the graph shown.



left 1 + up 3

y = □ | x + □ | + □

y = 2 | x + 1 | + 3

Name ______________________ Class ______________ Date ______________

Practice 2-6

Vertical and Horizontal Translations

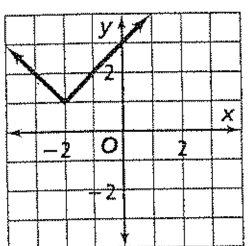
Describe each translation of $f(x) = |x|$ as vertical, horizontal, or diagonal. Then graph each translation.

1. $f(x) = |x + 2|$ H
2. $f(x) = |x + 4|$ H
3. $f(x) = |x| - 5$ V
4. $f(x) = |x + 1| - 1$ D
5. $f(x) = |x - 2| + 1$ D
6. $f(x) = \left|x - \frac{3}{2}\right|$ H
7. $f(x) = |x| - \frac{1}{3}$ V
8. $f(x) = \left|x - \frac{5}{2}\right|$ H
9. $f(x) = \left|x + \frac{1}{2}\right| + \frac{3}{2}$ D

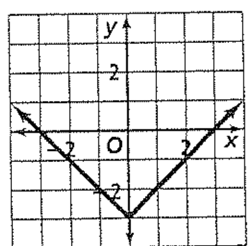
Write an equation for each translation.

10. $y = |x|$, 1 unit up, 2 units left $y=|x+2|+1$
11. $y = |x|$, 4 units right $y=|x-4|$
12. $y = -|x|$, 3 units up, 1 unit right $y=-|x-1|+3$
13. $y = -|x|$, $\frac{3}{2}$ units down, $\frac{1}{2}$ unit right $y=-|x-1/2|-3/2$
14. $y = |x|$, 2 units down, 3 units left $y=|x+3|-2$
15. $y = -|x|$, $\frac{3}{5}$ unit up $y=-|x|+3/5$

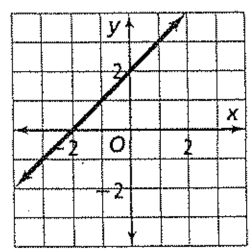
Write the equation of each translation of $y = x$ or $y = |x|$.

16.


$y=|x+2|+1$

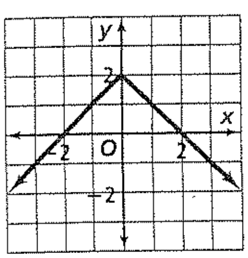
17.


$y=|x|-3$

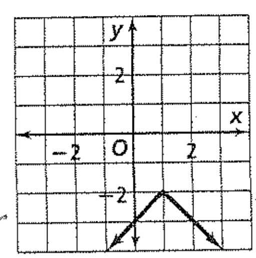
18.


$y=x+2$

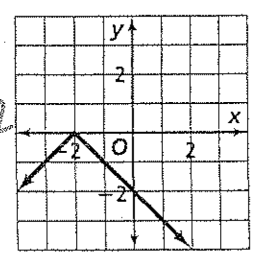
Each graph shows a translation of $y = -|x|$. State the values of h and k.

19.


$h=0$
$k=2$

20.


$h=1$
$k=-2$

21.


$h=-2$
$k=0$

Graph each equation.

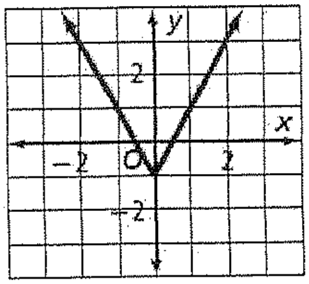
22. $y = |x - 1| + 2$
23. $y = -\left|x + \frac{1}{2}\right|$
24. $y = -|x + 3| - 1$
~~25. $y = |-x - 1|$~~ OMIT
26. $y = -|x - 2| + 4$
27. $y = |x + 2| - 1$

(12)

Unit 2 Lesson 6 - Absolute Value Transformations – HOMEWORK

Match each equation with its graph.

1. $y = |x - 1|$ E
2. $y = 2|x - 1|$ C
3. $y = |2x| - 1$ A
4. $y = |x| - 1$ F
5. $y = |2x - 1|$ B
6. $y = |2x| - 2$ D

A.


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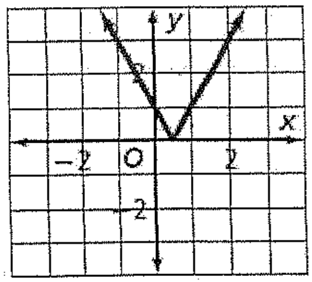

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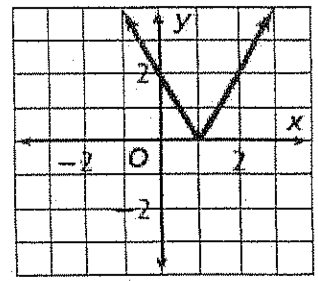
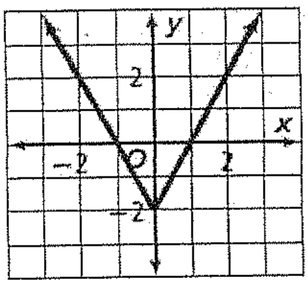


D.


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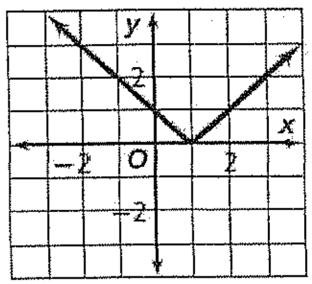

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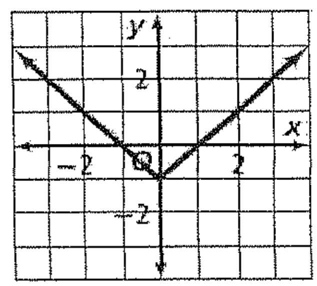


Describe each transformation of f(x) = |x| using Up, down, left, right, and the number of units.

1. $f(x) = |x + 2|$

left 2

2. $f(x) = |x + 4|$

left 4

3. $f(x) = |x| - 5$

down 5

4. $f(x) = |x + 1| - 1$

left 1, down 1

5. $f(x) = |x - 2| + 1$

right 2, up 1

6. $f(x) = \left|x - \frac{3}{2}\right|$

right 3/2

7. $f(x) = |x| - \frac{1}{3}$

down 1/3

8. $f(x) = \left|x - \frac{5}{2}\right|$

right 5/2

9. $f(x) = \left|x + \frac{1}{2}\right| + \frac{3}{2}$

left 1/2, up 3/2

Go to page 2

13

Warm Up

1. Solve for y: $5y - 5 = 3$ $\rightarrow \frac{5y}{5} = \frac{8}{5} \Rightarrow y = \frac{8}{5}$

2. Determine if the following is

cross products   reduce, common factor of 3

true: $\overset{30}{}\frac{2}{5} = \frac{6}{15}\overset{30}{}$ ✓ $\frac{2}{5} = \frac{\cancel{6}^{2}}{\cancel{15}_{5}}$ ✓   yes

3. Factor: $x^2 - 6x + 8$ -2, -4, 8, -6

$(x-2)(x-4)$

4. Simplify: $\left(4a^2b\right)^3\left(ab\right)^2 \Rightarrow 64a^6b^3 \cdot a^2b^2$

$\Rightarrow 64a^8b^5$

*5. Solve: $\frac{4x^2}{4} = \frac{-1}{4}$

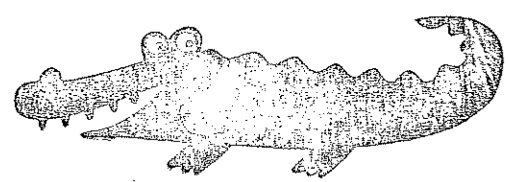
$\sqrt{x^2} = \sqrt{-\frac{1}{4}}$

$x = \frac{\sqrt{-1}}{2} = \frac{i}{2}$

* no real answer

Graphing Inverse Variations

$y = \frac{k}{x}$



A relationship that can be written in the form $y = k/x$, where k is a nonzero constant and $x \neq 0$, is an inverse variation

The constant k is the constant of variation

Inverse variation implies that one quantity will increase while the other quantity will decrease (the inverse, or opposite, of increase).

The domain is all real numbers except zero.

Why?

Since x is in the denominator, the only restriction is $x \neq 0$ since there is no division by 0.

$\{x \in \mathbb{R}, x \neq 0\}$

The range is all real numbers except zero.

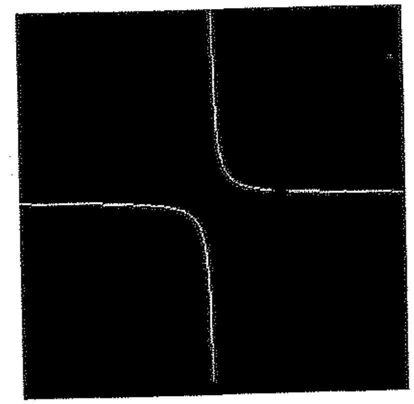
Why?

k is nonzero or it would not vary.
x is nonzero also.
Thus y will never be 0 either.

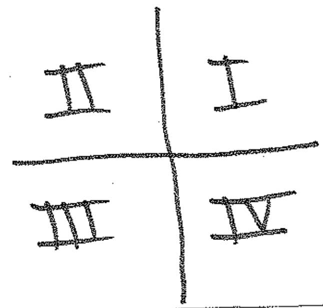
$\{y \in \mathbb{R}, y \neq 0\}$

The graphs of inverse variations have two parts.

Ex. f(x) = 1/x

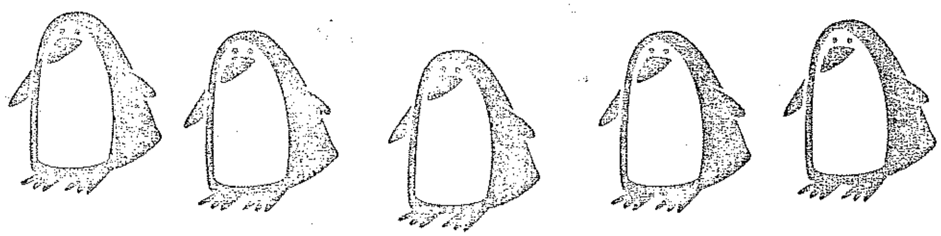


Each part is called a branch.



Since both the domain and range have restrictions at zero, the graph can never touch x or y axis

This creates asymptotes at the axis. "boundaries"



When k is positive, the branches are in Quadrants I + III.

When k is negative, the branches are in Quadrants II + IV.

Translations of Inverse Variations:

The graph of $y = \frac{k}{x-b} + c$

is a translation of y = k/x, b units horizontally and c units vertically.

The vertical asymptote is x=b.

The horizontal asymptote is y=c.

Translations of Inverse Variations: affects range "y"

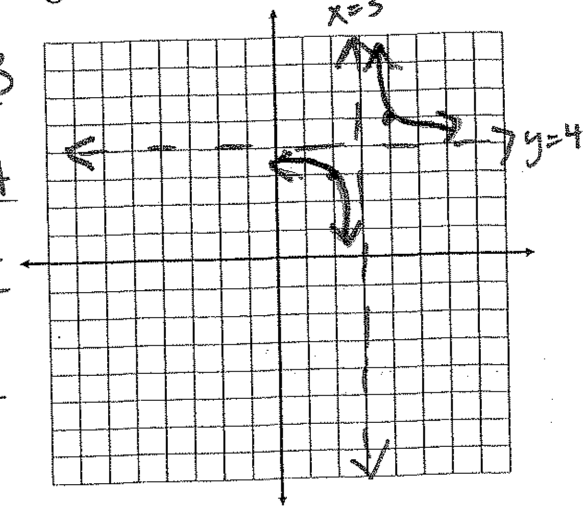
The graph of $y = \dfrac{k}{x-b} + c$ affects domain "x"

k tells us how far the branches have been stretched from asymptotes

We can use it to help us find out corner points to start our branches.

$\sqrt{k}$ = the distance from the asymptotes.

Example: $y = \dfrac{1}{x-3} + 4$



Vert. Asy: X=3

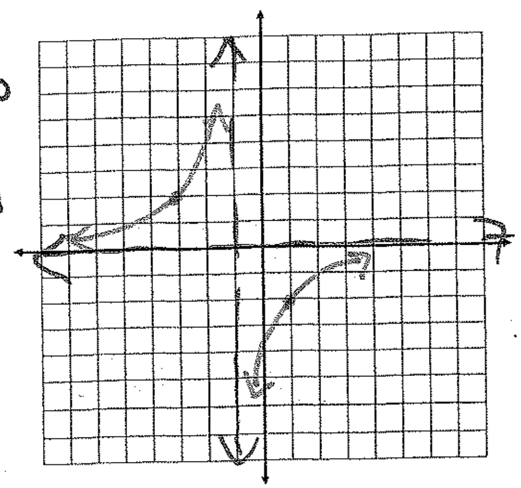
Horz. Asy: y=4

k=1 Quad: I & III

√1 = Distance: 1

You Try: Graph

$y = -\dfrac{4}{x+1} + 0$



Vert. Asy.: X=-1

Horz. Asy: y=0

k=-4 Quad: II & IV

√4 Distance: 2

We can also write the equation just given the parent function and the asymptotes.

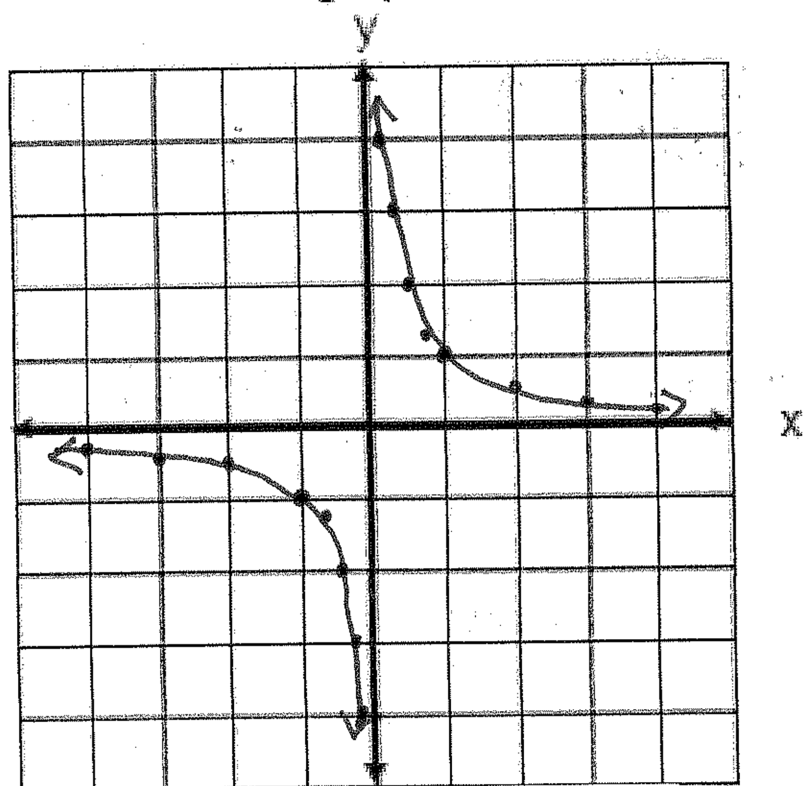
Ex. Write the equation of y = -1/x that has asymptotes x = -4 and y = 5.!

Answer: $y = \dfrac{-1}{x+4} + 5$

Inverse Variation

$f(x) = \frac{1}{x}$

Plot the points and sketch the graph below.



Complete the table of values.

x	f (x)
-4	$-1/4$
-3	$-1/3$
-2	$-1/2$
-1	-1
$-\frac{3}{4}$	$-4/3$
$-\frac{1}{2}$	-2
$-\frac{1}{3}$	-3
$-\frac{1}{4}$	-4
0	undefined
$\frac{1}{4}$	4
$\frac{1}{3}$	3
$\frac{1}{2}$	2
$\frac{3}{4}$	$4/3$
1	1
2	$1/2$
3	$1/3$
4	$1/4$

Why is this called an inverse variation? As one value increases, the other value decreases.

What is the end behavior of this function? It approaches zero.

Can x ever have a value of 0? no

Can f(x) ever have a value of 0? no

Properties
Of
Inverse Variation

$f(x) = \frac{1}{x}$

Domain:
$x \in \mathbb{R}$ except
$x \neq 0$

Range:
$y \in \mathbb{R}$ except
$y \neq 0$

Maximum:
$y = \infty$

Reflection:
$y = -x$

Minimum:
$y = -\infty$

Increasing: never

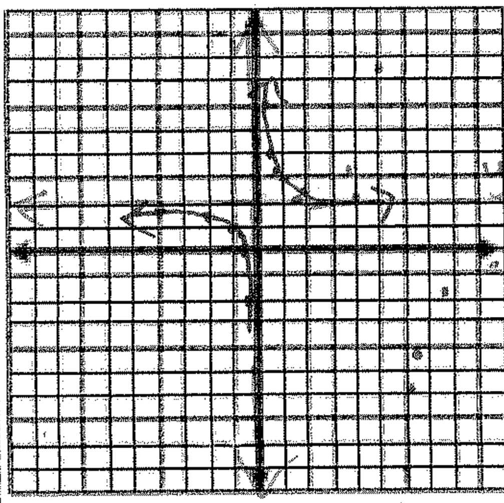
Decreasing: $(-\infty, 0)$ $-\infty < x < 0$
$(0, \infty)$ $\infty > x > 0$

What is the x-intercept?
none

What is the y-intercept?
none

$$y = \frac{1}{x} + 2$$

x	$f(x) = \frac{1}{x} + 2$	y
-4	$-\frac{1}{4} + 2$	$1\frac{3}{4}$
-2	$-\frac{1}{2} + 2$	$1\frac{1}{2}$
-1	$-1 + 2$	1
$-\frac{3}{4}$	$-\frac{4}{3} + 2$	$\frac{2}{3}$
$-\frac{1}{2}$	$-2 + 2$	0
$-\frac{1}{4}$	$-4 + 2$	-2
0	undefined	
$\frac{1}{4}$	$4 + 2$	6
$\frac{1}{2}$	$2 + 2$	4
* $\frac{3}{4}$	$\frac{4}{3} + 2$	$3\frac{1}{3}$
1	$1 + 2$	3
2	$\frac{1}{2} + 2$	$2\frac{1}{2}$
4	$\frac{1}{4} + 2$	$2\frac{1}{4}$



Domain: $x \in \mathbb{R}$ $x \neq 0$

Range: $y \in \mathbb{R}$ $y \neq 2$

Maximum/Minimum: $\infty / -\infty$

Increasing/Decreasing: all reals except 0

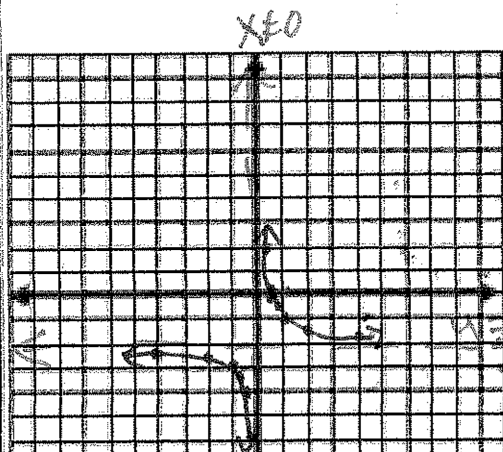
x-intercept: $\left(-\frac{1}{2}, 0\right)$

y-intercept: none

Transformation Type: translation up 2

RATIONAL FUNCTIONS

x	$f(x) = \frac{1}{x} - 2$	y
-4	$-\frac{1}{4} - 2$	$-2\frac{1}{4}$
-2	$-\frac{1}{2} - 2$	$-2\frac{1}{2}$
-1	$-1 - 2$	-3
$-\frac{3}{4}$	$-\frac{4}{3} - 2$	$-3\frac{1}{3}$
$-\frac{1}{2}$	$-2 - 2$	-4
$-\frac{1}{4}$	$-4 - 2$	-6
0	undefined	
$\frac{1}{4}$	$4 - 2$	2
$\frac{1}{2}$	$2 - 2$	0
* $\frac{3}{4}$	$\frac{4}{3} - 2$	$-\frac{2}{3}$
1	$1 - 2$	-1
2	$\frac{1}{2} - 2$	$-1\frac{1}{2}$
4	$\frac{1}{4} - 2$	$-1\frac{3}{4}$



This is also a rational function.

Domain: $x \in \mathbb{R}$, $x \neq 0$

Range: $y \in \mathbb{R}$, $y \neq -2$

Maximum/Minimum: $\infty / -\infty$

Increasing/Decreasing: all reals except 0

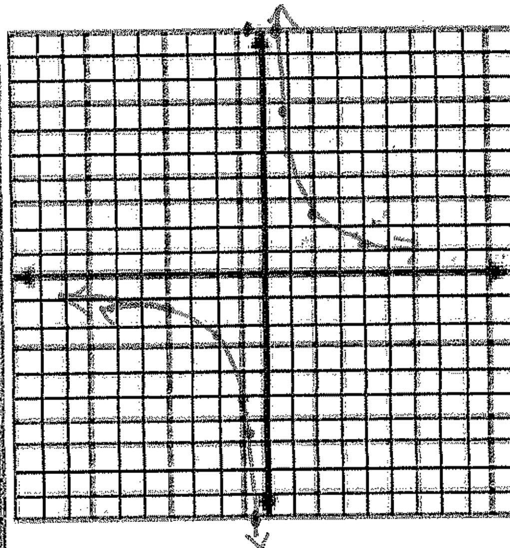
x-intercept: $\left(\frac{1}{2}, 0\right)$

y-intercept: none

Transformation Type: translation down 2

$$y = \frac{1}{x} - 2$$

x	$f(x) = 5 \cdot \frac{1}{x}$	y
-4	5 · 1/-4	-5/4
-2	5 · 1/-2	-5/2
-1	5 · -1	-5
$-\frac{3}{4}$	5 · -4/3	-6 2/3
$-\frac{1}{2}$	5 · -2	-10
$-\frac{1}{4}$	5 · -4	-20
0	undefined	
$\frac{1}{4}$	5 · 4	20
$\frac{1}{2}$	5 · 2	10
* $\frac{3}{4}$	$5 \cdot \frac{4}{3}$	6 2/3
1	5 · 1	5
2	5 · 1/2	5/2
4	5 · 1/4	5/4



Domain: x ∈ ℝ except x≠0

Range: y ∈ ℝ except y≠0

Maximum/Minimum: ∞ / -∞

Increasing/Decreasing

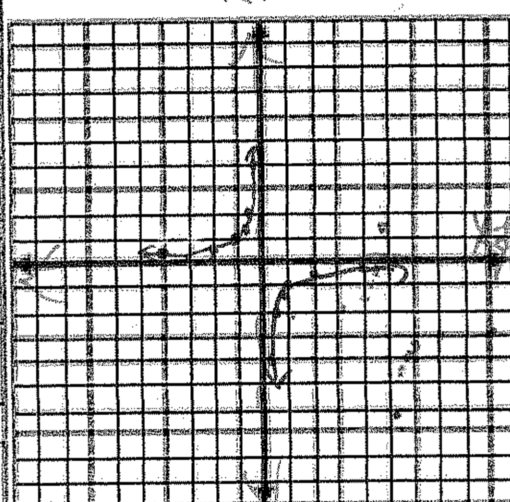
x-intercept: none

y-intercept: none

Transformation Type:
dilation

x	$f(x) = -\frac{1}{x}$	y
-4	-1/-4	1/4
-2	-1/-2	1/2
-1	-1/-1	1
$-\frac{3}{4}$	-1/-3/4	4/3
$-\frac{1}{2}$	-1/-1/2	2
$-\frac{1}{4}$	-1/-1/4	4
0	undefined	
$\frac{1}{4}$	-1/1/4	-4
$\frac{1}{2}$	-1/1/2	-2
* $\frac{3}{4}$	-1/3/4	-4/3
1	-1/1	-1
2	-1/2	-1/2
4	-1/4	-1/4

X≠0



This is also a rational function.

Domain: X ∈ ℝ except x≠0

Range: y ∈ ℝ except y≠0

Maximum/Minimum: ∞ / -∞

Increasing/Decreasing all numbers except 0

x-intercept: none

y-intercept: none

Transformation Type:
reflection x axis

6.9 Homework: Graphing Inverse Variation Functions

1. **VOCABULARY** Copy and complete: The function $y = \frac{7}{x+4} + 3$ has a(n) ___?___ of all real numbers except 3 and a(n) ___?___ of all real numbers except −4.

range

domain

2. ★ **WRITING** Is $f(x) = \frac{-3x+5}{2^x+1}$ a rational function? *Explain* your answer.

GRAPHING FUNCTIONS **Graph the function. Compare the graph with the graph of $y = \frac{1}{x}$.**

3. $y = \frac{3}{x}$
4. $y = \frac{10}{x}$
5. $y = \frac{-5}{x}$
6. $y = \frac{-0.5}{x}$
7. $y = \frac{0.1}{x}$
8. $f(x) = \frac{15}{x}$
9. $g(x) = \frac{-6}{x}$
10. $h(x) = \frac{-3}{x}$

Graph the function. State the domain, range, the x- and y-intercept(s) and the transformation.

11. $y = \frac{4}{x} + 3$
12. $y = \frac{3}{x} - 2$
13. $y = \frac{6}{x-1}$
14. $f(x) = \frac{1}{x+2}$
15. $y = \frac{-5}{x} - 7$
16. $y = \frac{-6}{x} + 4$
17. $y = \frac{-3}{x+2}$
18. $g(x) = \frac{-2}{x-7}$
19. $y = \frac{-4}{x+4} + 3$
20. $y = \frac{10}{x+7} - 5$
21. $y = \frac{-3}{x-4} - 1$
22. $h(x) = \frac{11}{x-9} + 9$

Name ______________________ Class ______________ Date ______________

Practice 9-2

Graphing Inverse Variations

Write an equation for a translation of $y = -\frac{3}{x}$ that has the given asymptotes.

1. $x = 2; y = 1$
2. $x = -1; y = 3$
3. $x = 4; y = -2$
4. $x = 0; y = 6$
5. $x = 3; y = 0$
6. $x = 1; y = 2$
7. $x = -3; y = -1$
8. $x = -2; y = 1$

Sketch the asymptotes and the graph of each equation.

9. $y = \frac{3}{x-1} + 2$
10. $y = \frac{2}{x+1}$
11. $y = \frac{11}{x+3} - 3$
12. $y = -\frac{4}{x-2} - 2$
13. $y = \frac{1}{x} + 3$
14. $y = \frac{1}{x+1} - 2$
15. $y = \frac{1}{x-2} + 1$
16. $y = \frac{1}{x-1} - 1$
17. $y = \frac{2}{x}$
18. $y = -\frac{3}{x-3} + 1$
19. $y = \frac{1}{x+1} + 2$
20. $y = \frac{3}{4x} + \frac{1}{2}$
21. $y = \frac{3}{x+3} - 1$
22. $y = \frac{2}{x-5}$
23. $y = -\frac{6}{x-3} - 2$
24. $y = \frac{5}{x}$
25. $y = \frac{1}{x-1} + 1$
26. $y = \frac{1}{x}$
27. $y = -\frac{3}{x-4} - 2$
28. $y = -\frac{1}{x-2} - \frac{1}{2}$

The length of a panpipe p (in feet) is inversely proportional to its pitch ℓ (in hertz). The inverse variation is modeled by the equation $p = \frac{495}{\ell}$.

29. Find the length required to produce a pitch of 220 Hz.
30. What pitch would be produced by a pipe with a length of 1.2 ft?
31. Find the pitch of a 0.6-ft pipe.
32. Find the pitch of a 3-ft pipe.

The junior class is buying keepsakes for the junior-senior prom. The price of each keepsake p is inversely proportional to the number of keepsakes s bought. The equation $p = \frac{1800}{s}$ models this inverse variation.

33. If they buy 240 keepsakes, how much can the class spend for each?
34. If they spend \$5.55 for each keepsake, how many can the class buy?
35. If 400 keepsakes are bought, how much can be spent for each?
36. If the class buys 50 keepsakes, how much can be spent for each?

Compare the graphs of the inverse variations.

37. $y = \frac{1}{x}$ and $y = \frac{5}{x}$
38. $y = \frac{3}{x}$ and $y = -\frac{3}{x}$
39. $y = \frac{2}{x}$ and $y = \frac{20}{x}$
40. $y = -\frac{1}{x}$ and $y = -\frac{10}{x}$
41. $y = \frac{6}{x}$ and $y = -\frac{6}{x}$
42. $y = \frac{0.2}{x}$ and $y = \frac{0.02}{x}$

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Warm-Up: Solving Radical Equations

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. If x^2 equals 25, then x must equal what?
   - a. 5
   - b. -5
   - c. ± 5
   - d. $\sqrt{5}$

2. Which of these is equivalent to $8^{\frac{1}{3}}$?
   - a. $2.\overline{3}$
   - b. 2
   - c. $\frac{1}{2}$
   - d. $\frac{1}{8}$

3. If $\sqrt{x}$ equals 5, then x must equal what?
   - a. 25
   - b. ± 25
   - c. $\sqrt{5}$
   - d. $\pm\sqrt{5}$

4. If $\sqrt{a}$ equals 1, then a must equal what?
   - a. -1
   - b. 1
   - c. ± 1
   - d. no solution

5. If $16^?$ equals 4?
   - a. -2
   - b. $\frac{1}{2}$
   - c. -4
   - d. $\frac{1}{4}$

Graphing Square and Cube Root Functions

Make a table for each function.

$F(x) = x^2$

x	f(x)
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81

$f(x) = \sqrt{x}$

x	f(x)
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$
4	2
5	$\sqrt{5}$
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	3

$f(x) = x^3$

x	f(x)
-8	-512
-6	-216
-4	-64
-2	-8
-1	-1
0	0
1	1
2	8
4	64
6	216
8	512

$f(x) = \sqrt[3]{x}$

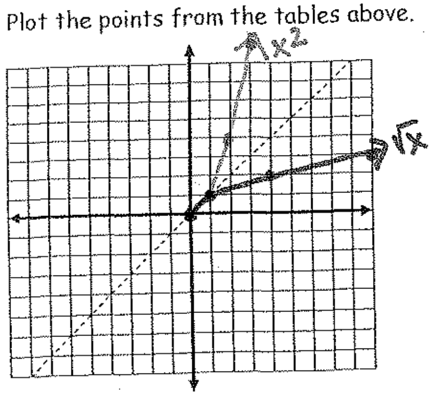
x	f(x)
-8	-2
-6	-1.82
-4	-1.59
-2	-1.26
-1	-1
0	0
1	1
2	1.26
4	1.59
6	1.82
8	2

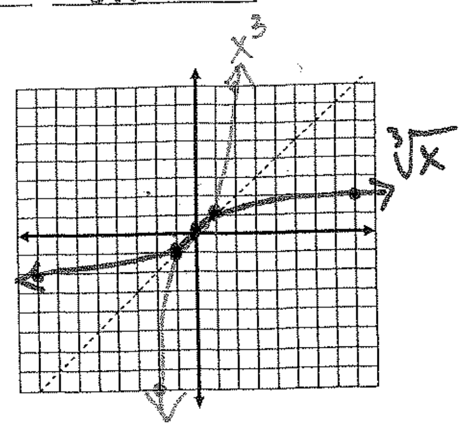
Ignore the points with decimals. What do you notice about the other points?

symmetry

These functions are inverses of each other. By definition, this means the domain and the range switch.

Plot the points from the tables above.



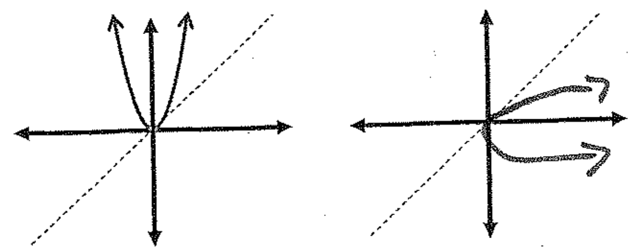


As a result, the graphs have the same numbers in their points but the x and the y coordinates have switched places.

This causes the graphs to have the same shape but to be reflected over the line y=x.

The Square Root Function

Reflect the function $f(x) = x^2$ over the line $y = x$.

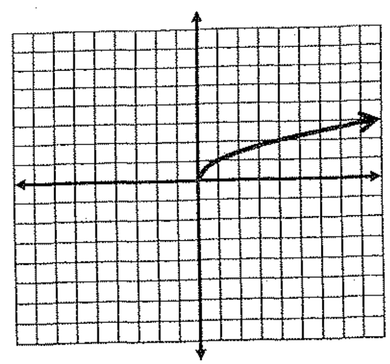


Problems? not a function

We have to define the Square Root function as $\sqrt{x^2} = |x|$.

This means that we will only use the positive (top) side of the graph.

The result: $f(x) = \sqrt{x}$



Characteristics of the graph

Vertex (0,0)

End Behavior As x increases $y \to \infty$
As x decreases $y \to 0$

Domain $x \geq 0$

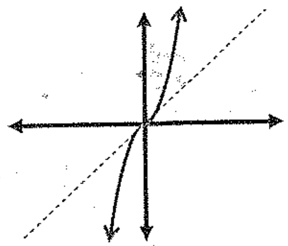
Range $y \geq 0$

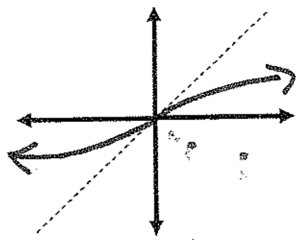
Symmetry NONE

Pattern NONE

The Cube Root Function

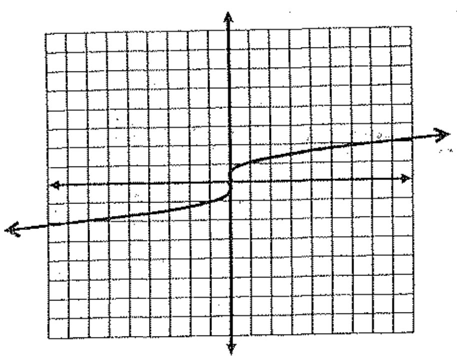
Reflect the function $f(x) = x^3$ over the line $y = x$.





Problems? NONE

The result: $f(x) = \sqrt[3]{x}$



Characteristics of the graph

Vertex (0,0)

End Behavior As x increases, y increases $y \to \infty$
As x decreases, y decreases $y \to -\infty$

Domain $x \in \mathbb{R}$

Range $y \in \mathbb{R}$

Symmetry rotational

Pattern

Transforming the Graphs

Now that we know the shapes we can use what we know about transformations to put that shape on the coordinate plane.

Remember:

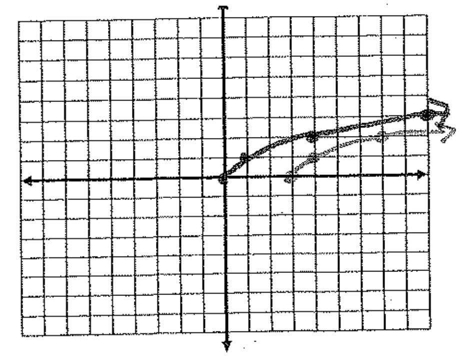
Translate	Reflect	Dilate
$y = \sqrt{x+1}$ left 1	$y = -\sqrt{x}$	$y = 2\sqrt{x}$
$y = \sqrt{x-2}$ right 2	reflects x axis	vertical stretch
$y = \sqrt{x} + 3$ up 3	$y = \sqrt{-x}$	$y = \frac{1}{4}\sqrt{x}$
$y = \sqrt{x} - 4$ down 4	reflects y axis	vertical compression

Describe transformation

1) $f(x) = \sqrt{x - 3}$

right 3

Parent: $\sqrt{x}$

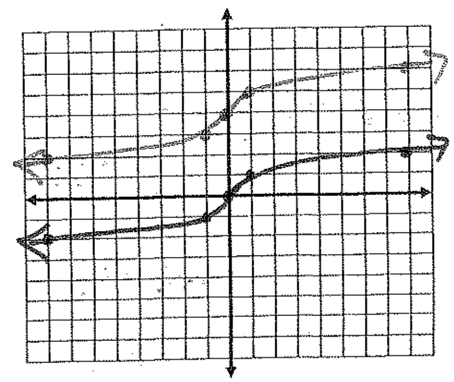


2) $f(x) = \sqrt[3]{x} + 4$

up 4

Parent: $\sqrt[3]{x}$

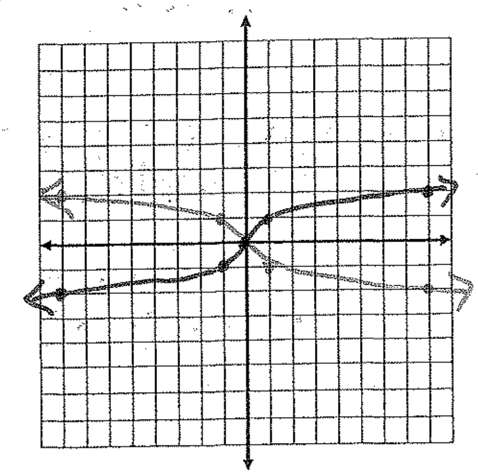
(0,0)
(1,1)
(8,2)
(-1,-1)
(-8,-2)



3) $f(x) = -\sqrt[3]{x}$

reflect x axis

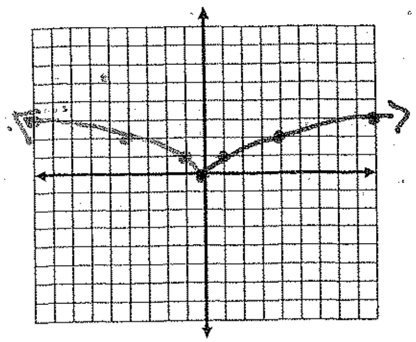
Parent: $\sqrt[3]{x}$



4) $f(x) = \sqrt{-x}$

reflects y axis

Parent: $\sqrt{x}$

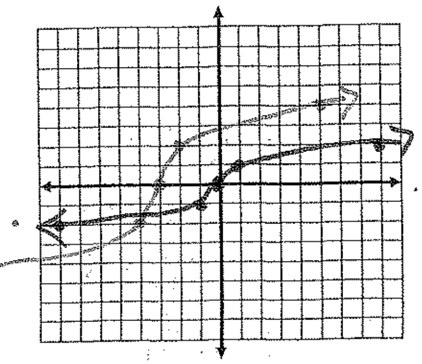


5) $f(x) = 2\sqrt[3]{x+3}$

Parent $\sqrt[3]{x}$

left 3

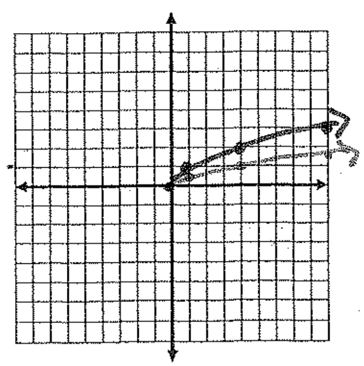
multiply by 2



6) $f(x) = \frac{1}{2}\sqrt{x}$

Parent: $\sqrt{x}$

multiply by $\frac{1}{2}$



Sometimes the functions are not in graphing form. We may have to use some of our algebra skills to transform the equations into something we can use.

Ex: $f(x) = \sqrt{4x-12}$ This is not in graphing form.

$f(x) = \sqrt{4(x-3)}$ Factor out 4.

$f(x) = \sqrt{4} \cdot \sqrt{x-3}$ Divide

$f(x) = 2\sqrt{x-3}$ Simplify

$\boxed{\text{vertical stretch + right 3}}$ ← Transformation

Ex: $f(x) = \sqrt[3]{8x+32} - 5$ This is not in graphing form.

$f(x) = \sqrt[3]{8(x+4)} - 5$ Factor out 8

$f(x) = \sqrt[3]{8}\,\sqrt[3]{x+4} - 5$ Divide

$f(x) = 2\sqrt[3]{x+4} - 5$ Simplify

vertical stretch: 2 left 4 down 5

← Transformation

Algebra 2

Name

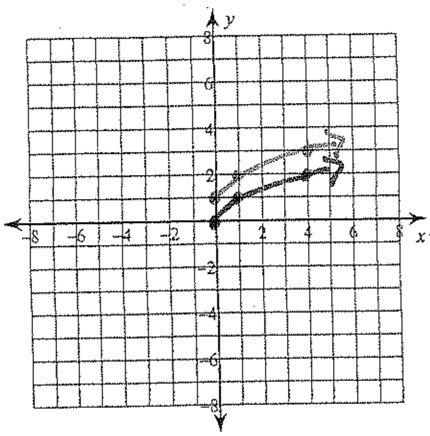
Square Root & Cube Root Guided Pratice

Identify the domain and range of each. Then sketch the graph.

1) $y = 1 + \sqrt{x}$

Parent: $\sqrt{x}$

up 1

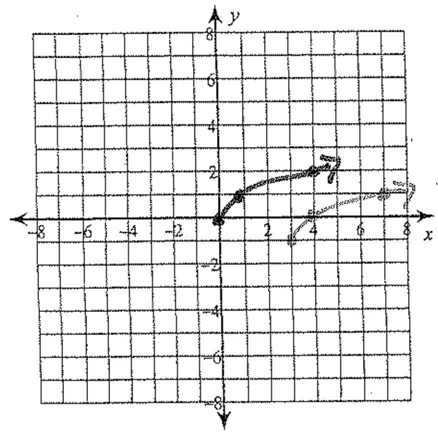


2) $y = \sqrt{x - 3} - 1$

Parent: $\sqrt{x}$

down 1

right 3

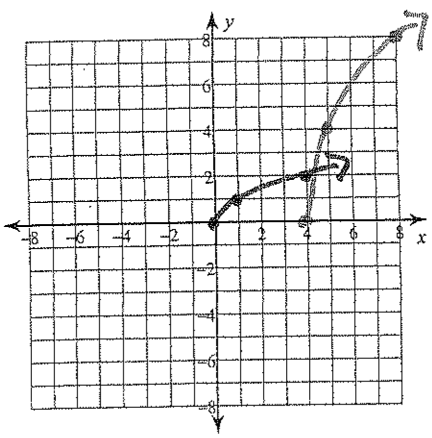


3) $y = 3\sqrt{x - 4}$

Parent: $\sqrt{x}$

right 4

vertical stretch of 3



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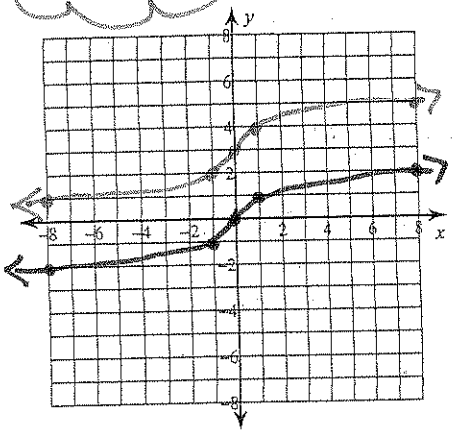
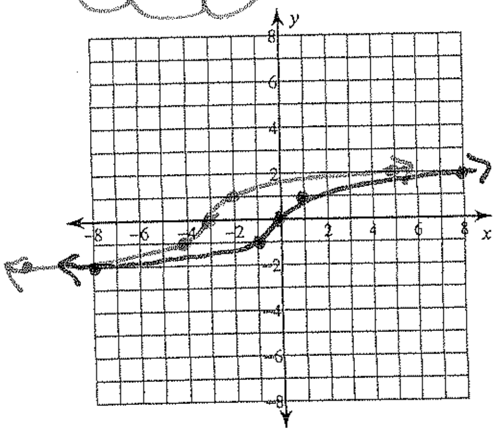
4) $y = \sqrt[3]{x} + 3$



Parent: $\sqrt[3]{x}$

up 3

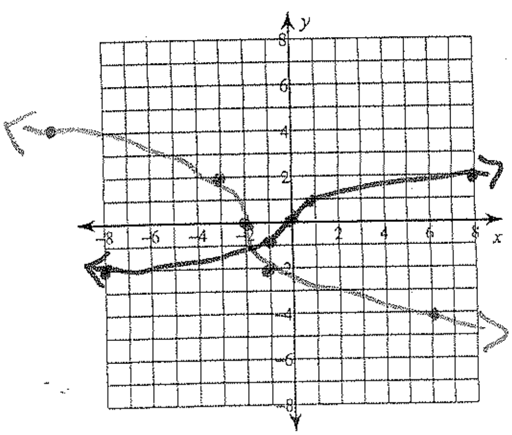
5) $y = \sqrt[3]{x+3}$



Parent: $\sqrt[3]{x}$

left 3

6) $y = -2\sqrt[3]{x+2}$



Parent $\sqrt[3]{x}$

left 2

reflect

multiply by 2

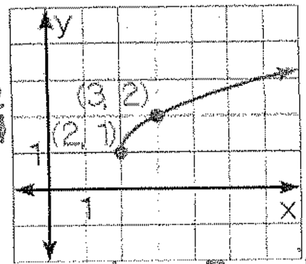
(29)

6.8 Practice: Graphing Square Root and Cube Root Functions

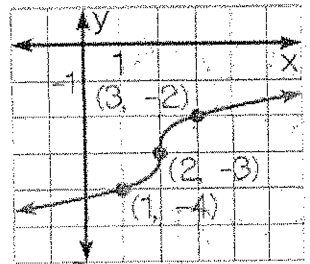
1. Complete this statement: Square root functions and cube root functions are examples of _?_ functions.

2. **ERROR ANALYSIS** Explain why the graph shown at the near right is not the graph of $y = \sqrt{x-1} + 2$. It went right 2 and up 1 instead of right 1 and up 2.

3. **ERROR ANALYSIS** Explain why the graph shown at the far right is not the graph of $y = \sqrt[3]{x+2} - 3$. It went right 2 & down 3 instead of left 2 & down 3.



Ex. 2



Ex. 3

Describe how to obtain the graph of *g* from the graph of *f*.

4. $g(x) = \sqrt{x+5}$, $f(x) = \sqrt{x}$ left 5

5. $g(x) = \sqrt[3]{x} - 10$, $f(x) = \sqrt[3]{x}$ down 10

Graph the function. Then state the domain and range.

6. $y = -\sqrt{x}$

7. $y = \sqrt{x+1}$

8. $y = \sqrt{x-2}$

9. $y = 2\sqrt{x+3} - 1$

10. $y = \frac{2}{3}\sqrt[3]{x}$

11. $y = \sqrt[3]{x} - 6$

12. $y = \sqrt[3]{x+5}$

13. $y = -3\sqrt[3]{x-7} - 4$

COMPARING GRAPHS **Describe how to obtain the graph of *g* from the graph of *f*.**

15. $g(x) = \sqrt{x+14}$, $f(x) = \sqrt{x}$ left 14

16. $g(x) = 5\sqrt{x-10} - 3$, $f(x) = 5\sqrt{x}$ right 10 down 3

17. $g(x) = -\sqrt[3]{x} - 10$, $f(x) = -\sqrt[3]{x}$ down 10

18. $g(x) = \sqrt[3]{x+6} - 5$, $f(x) = \sqrt[3]{x}$ left 6 down 5

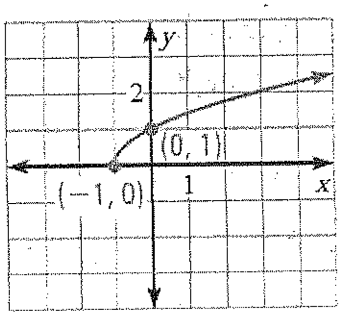
MATCHING GRAPHS **Match the function with its graph.**

19. $y = \sqrt{x} - 1$ B

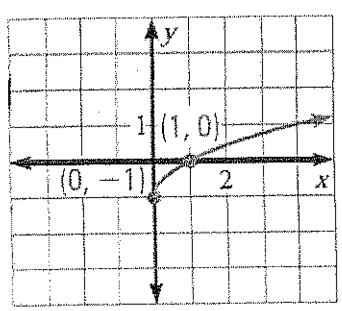
20. $y = \sqrt{x+1}$ A

21. $y = \sqrt{x+1} - 1$ C

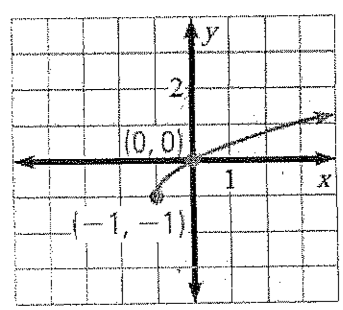
A.



B.



C.



Common Core 2

Name________________________________

Graphing Square and Cube Root Functions

ODD: Identify the domain and range of each. Then sketch the graph.
EVEN: Describe transformation from parent

1) $y = \sqrt{x+4}$

D: $x \geq -4$

R: $y \geq 0$

2) $y = -\sqrt{x} - 3$

down 3
reflect x axis

3) $y = 3 + \sqrt{x+3}$

D: $x \geq 3$

R: $y \geq 3$

4) $y = -\sqrt{x-1} - 3$

down 3
right 1
reflect x axis

5) $y = \sqrt{x}$

D: $x \geq 0$

R: $y \geq 0$

6) $y = \sqrt{x-2} + 1$

up 1
right 2

7) $y = \sqrt{x-1}$

D: $x \geq 1$

R: $y \geq 0$

8) $y = \sqrt{x-2} + 2$

up 2
right 2

9) $y = 3 + \sqrt{x-2}$

D: $x \geq 2$

R: $y \geq 3$

10) $y = \sqrt{x} + 4$

up 4

11) $y = -5 + \sqrt[3]{x}$

D: $x \geq 0$

R: $y \geq -5$

12) $y = \sqrt[3]{x-3}$

right 3

13) $y = \sqrt[3]{x}$

D: $x \geq 0$

R: $y \geq 0$

14) $y = \sqrt[3]{x+4}$

left 4

15) $y = \sqrt[3]{x+3}$

D: $x \geq -3$

R: $y \geq 0$

16) $y = \sqrt[3]{x-2} - 3$

right 2
down 3

17) $y = \sqrt[3]{x+3} - 1$

D: $x \geq -3$

R: $y \geq -1$

18) $y = \sqrt[3]{64x} = \sqrt[3]{64}\sqrt[3]{x}$
$= 4\sqrt[3]{x}$

vertical stretch of 4

19) $y = \sqrt[3]{x} + 5$

D: $x \geq 0$

R: $y \geq 5$

20) $y = 2\sqrt[3]{x-4}$

right 4
vertical stretch of 2

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