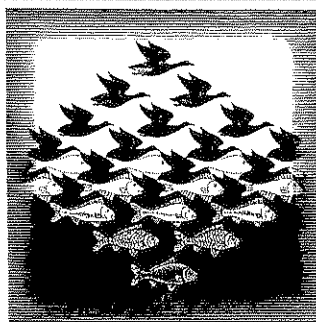


UNIT 3: Similarity and Congruence		HOMework
10/2 Wednesday	Sum of Angles and Triangle Theorems Pages 1-5	
10/3 Thursday	Equilateral and Isosceles Triangles Pages 6-10	
10/4 Friday	Similar Polygons Pages 11-14	
10/7 Monday	Similarity Pages 15-18	<b>QUIZ</b>
10/8 Tuesday	Mid-segment of a Triangle Pages 19-23	
10/9 Wednesday	Congruence and Rigid Motions Pages 24-27	
10/10 Thursday	Congruence Postulates -SSS, SAS, ASA, AAS, HL Corresponding Parts Pages 28-34	
<del>10/11</del> 10/11 Friday	Review and Vocabulary Pages 35-38	Study for Test
<del>10/14</del> 10/14 Friday	MON	
<del>10/15</del> 10/15 TUES	Unit 3 Test	
<del>10/16</del> 10/16 Monday	WED	
	PSAT	

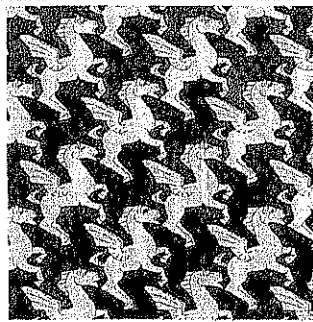
*FRIDAY - QUIZ (Similar Figures + Midsegments)*



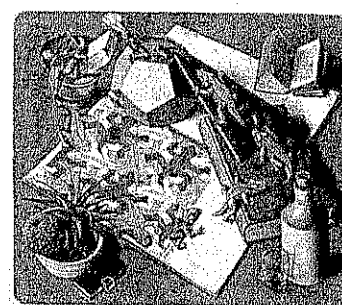
Lizard



Sky and Water



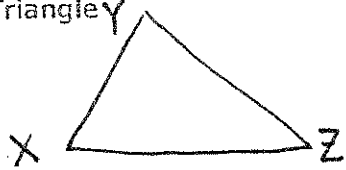
Pegasus



Reptiles

Works by M. C. Escher

Parts of Any Triangle



isosceles  
 equilateral  
 scalene  
 irregular

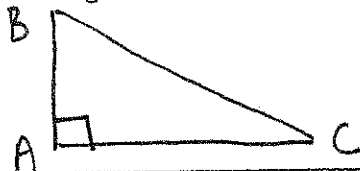
obtuse  
 acute  
 right  
 regular

sides  
 $\overline{XZ}, \overline{XY}, \overline{YZ}$

angles  
 $\angle X, \angle Y, \angle Z$

equiangular

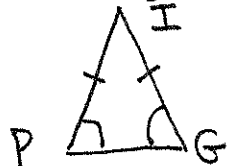
Parts of a Right Triangle



legs  
 $\overline{AB}, \overline{AC}$

hypotenuse  
 $\overline{BC}$

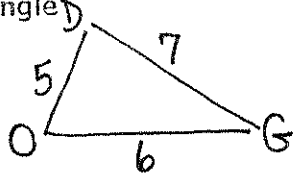
Parts of an Isosceles Triangle



at least two sides are equal

legs  $\overline{PI}, \overline{IG}$   
base  $\overline{PG}$   
vertex angle  $\angle I$   
base angles  $\angle P + \angle G$

Angles of a Triangle

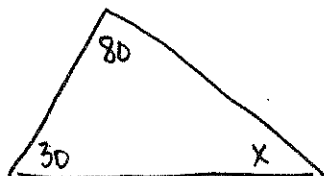


In a triangle, the longest side is opposite the largest angle.

Put the angles in order least to greatest  $\angle G, \angle D, \angle O$

Triangle Sum Theorem: The sum of the measures of the interior angles of a triangle is 180.

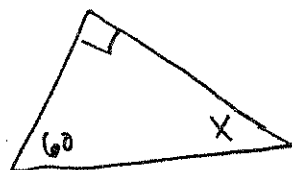
Example:



$$\underline{30} + \underline{80} + \underline{x} = 180$$

Corollary to Triangle Sum Thm: The acute angles of a right triangle are complementary.

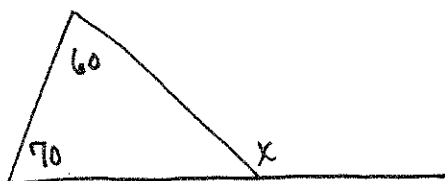
Example:



$$\underline{60} + \underline{x} = 90$$

Exterior Angles Theorem: The measure of an exterior angle of a triangle is equal to the sum of the measures of the 2 remote interior angles.

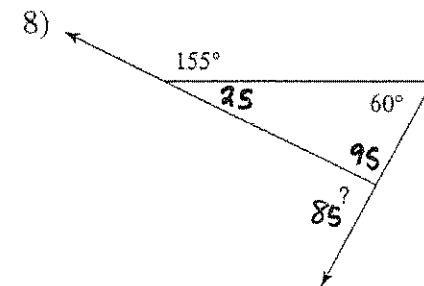
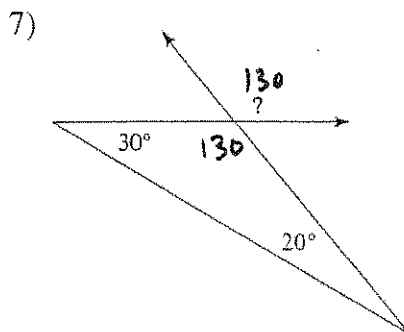
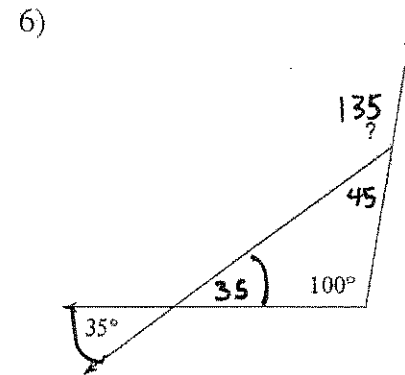
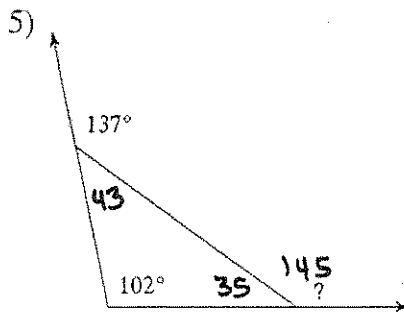
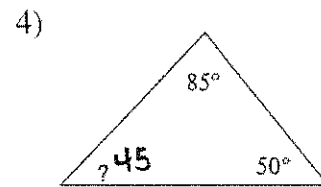
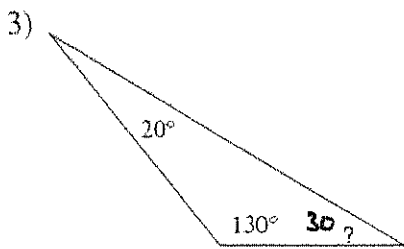
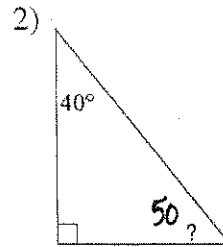
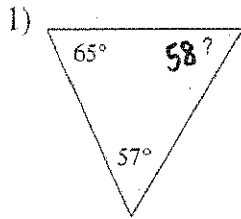
Example:



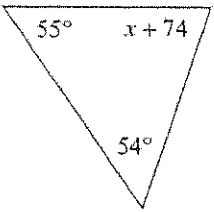
$$x = \underline{60} + \underline{70}$$

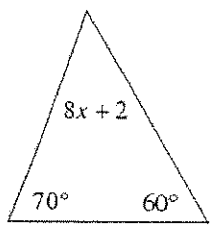
# Angles in a Triangle

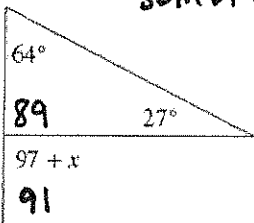
Find the measure of each angle indicated.

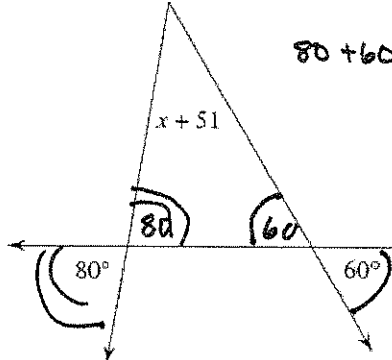


Solve for x.

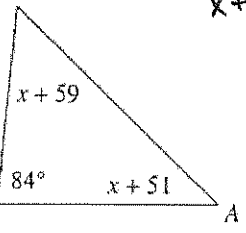
17)   $55 + 54 + x + 74 = 180$   
 $x = -3$

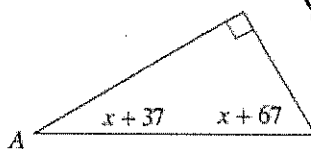
18)   $70 + 60 + 8x + 2 = 180$   
 $8x = 48$   
 $x = 6$

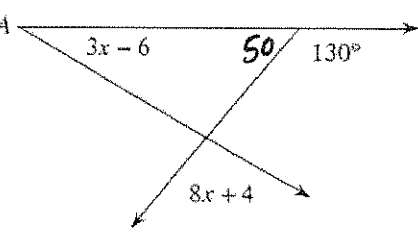
19)   $64 + 27 = 97 + x$   
 $-6 = x$   
 $97 + x = 91$   
 $x = -6$   
 sum of remote interiors = exterior angle  
 linear pairs  
 $89 + 97 + x = 180$

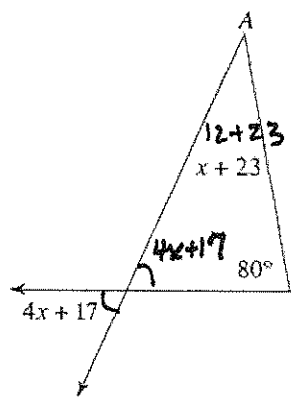
  $80 + 60 + x + 51 = 180$   
 $x = -11$

Find the measure of angle A.

21)   $x + 59 + x + 51 + 84 = 180$   
 $2x = -14$   
 $x = -7$   
 $m\angle A = \underline{\hspace{2cm}}$

22)   $x + 37 + x + 67 = 90$   
 $2x = -14$   
 $x = -7$   
 $m\angle A = \underline{\hspace{2cm}}$

23)   $8x + 4 + 50 + 3x - 6 = 180$   
 $11x = 132$   
 $x = 12$

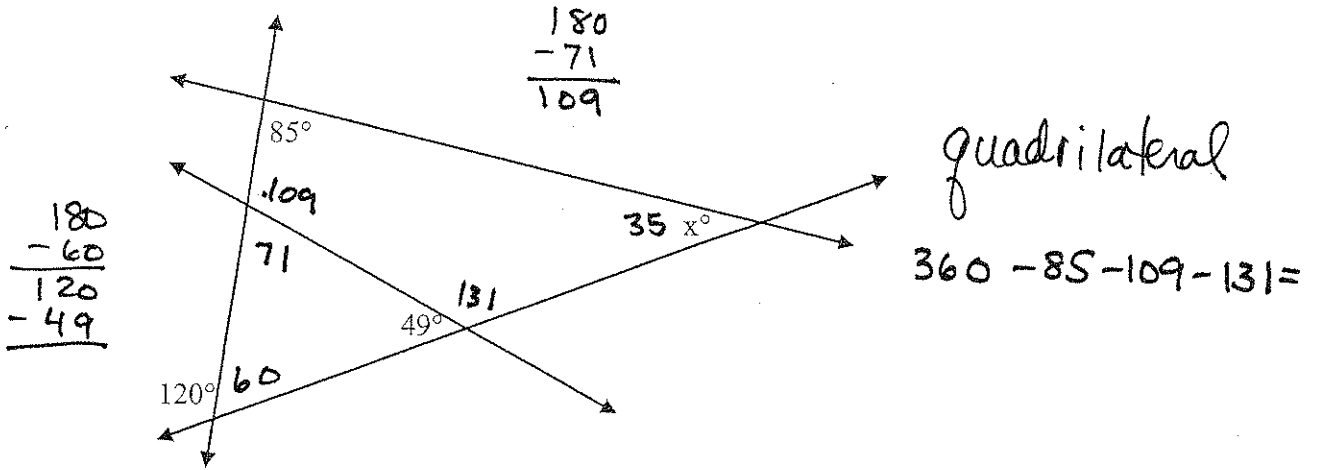
24)   $4x + 17 + x + 23 + 80 = 180$   
 $5x = 60$   
 $x = 12$   
 $m\angle A = \underline{35}$   
 $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = 180$

$m\angle A = \underline{\hspace{2cm}}$

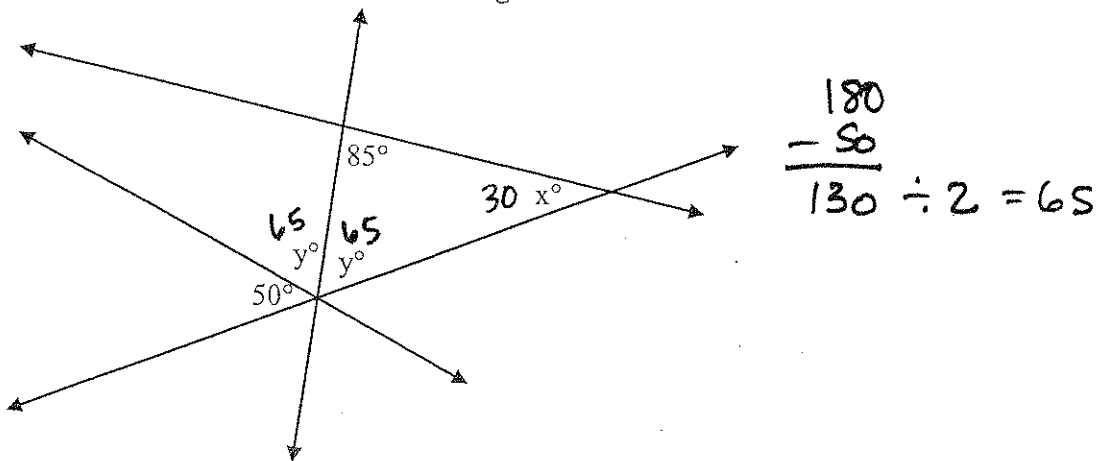
3

**Part 3: Challenge Problems**

10. Determine the measure of the unknown angle.

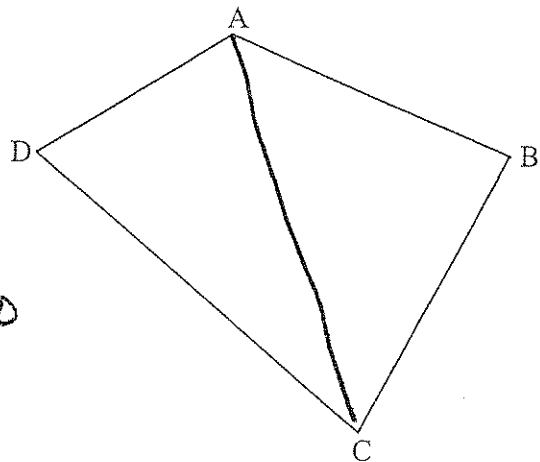


11. Determine the measures of the unknown angles.

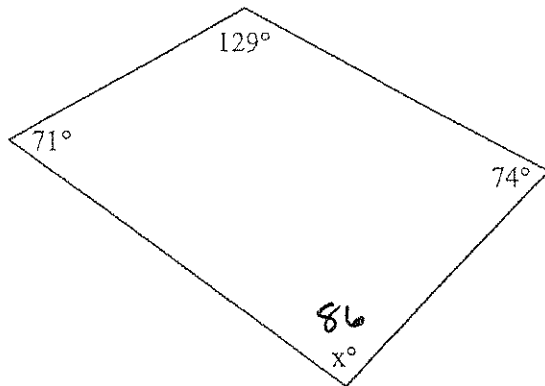


12. Consider the quadrilateral (4-sided polygon) at right.

- On the figure, draw an auxiliary line from A to C.
- How many triangles are formed? **2**
- How many degrees, total, are in the interior angles of each triangle? **180**
- Make a conjecture: what is the sum of the interior angles in every quadrilateral? **360**

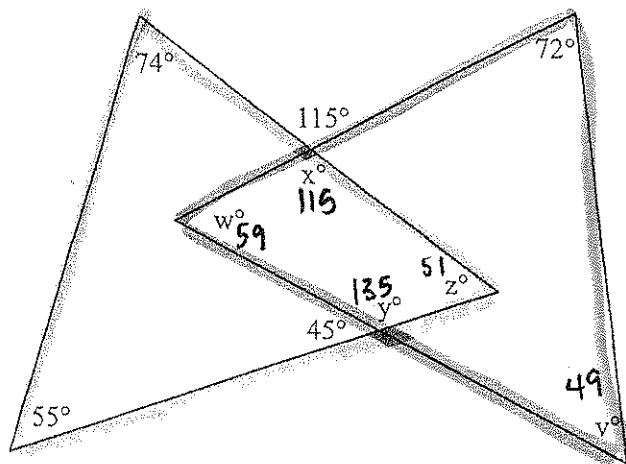


13. Determine the value of the unknown.



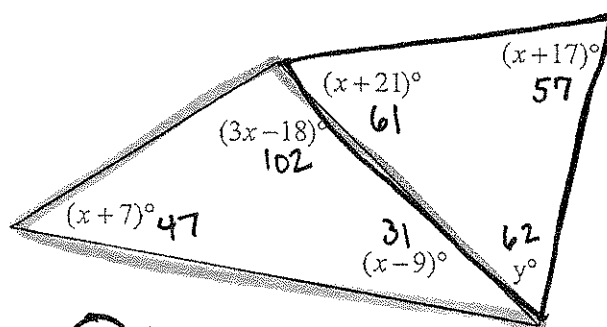
$$x = 86$$

14. Determine the value of the unknowns.



$$\begin{aligned} v &= 49 \\ w &= 59 \\ x &= 115 \\ y &= 135 \\ z &= 51 \end{aligned}$$

15. Determine the values of the unknown variables.



$$\begin{aligned} x &= 40 \\ y &= 62 \\ 61 + 57 + y &= 180 \end{aligned}$$

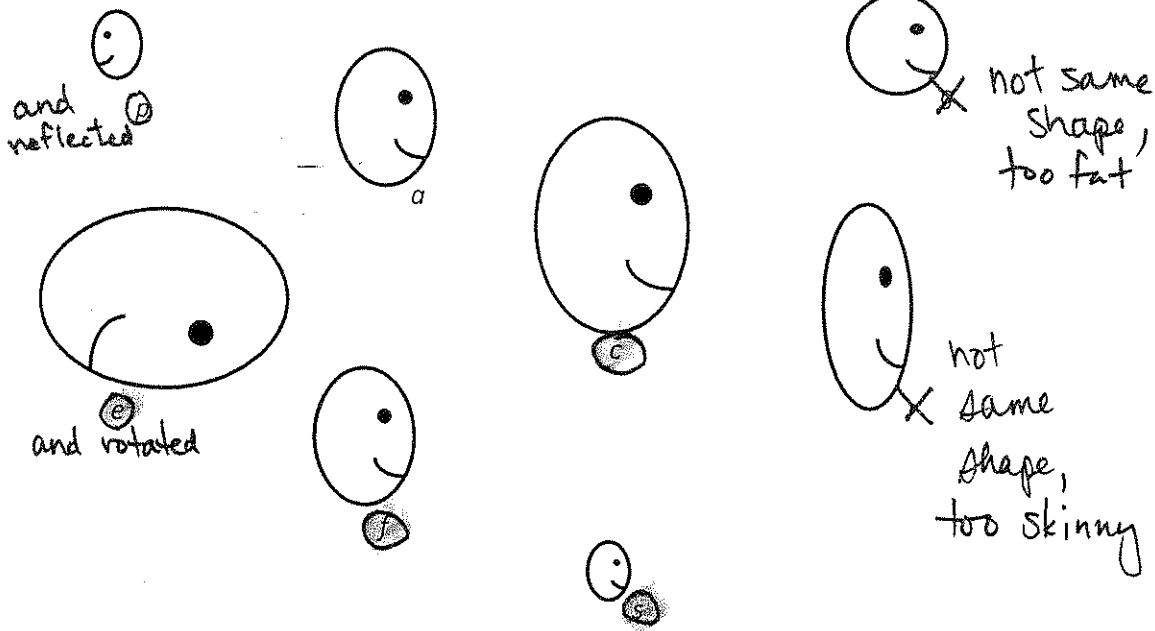
$$\begin{aligned} x + 7 + 3x - 18 + x - 9 &= 180 \\ 5x - 20 &= 180 \\ 5x &= 200 \\ x &= 40 \end{aligned}$$

$$\begin{aligned} x + 21 + x + 17 + y &= 180 \\ 40 + 21 + 40 + 17 + y &= 180 \\ y &= 62 \end{aligned}$$

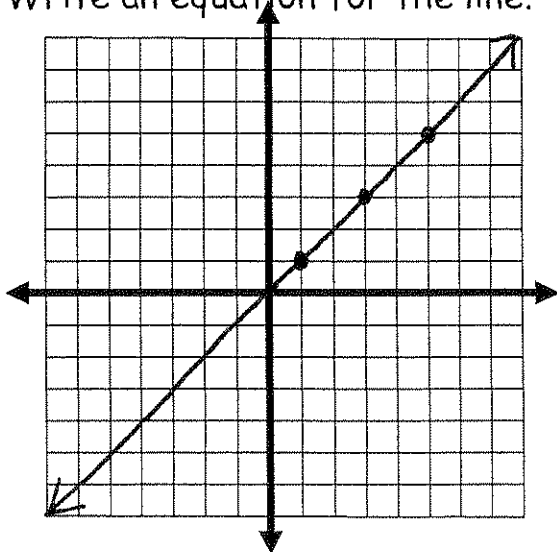
5

### 3.2 Warm Up

1. Which of the figures below could be the image of figure a when dilated? Explain why or why not for each figure.



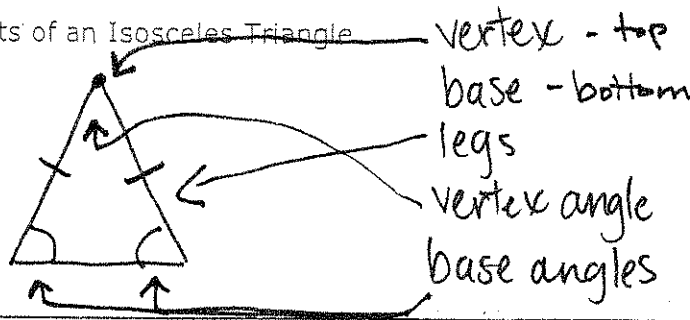
- 2) a) Draw a line that passes through the origin of a coordinate plane and forms a  $45^\circ$  angle with the x-axis.  
 b) Find the coordinates of at least three points on the line.  $(1,1)$   $(3,3)$   $(5,5)$   
 c) Write an equation for the line. What do you notice?



$$y = x$$

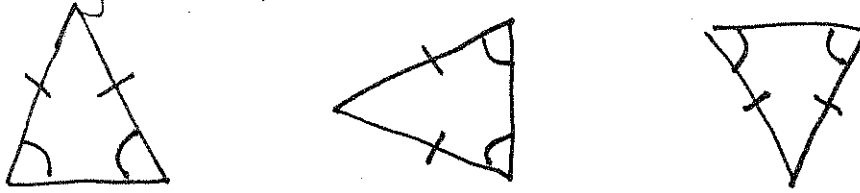
The x & y coordinates are the same.

Review: Parts of an Isosceles Triangle



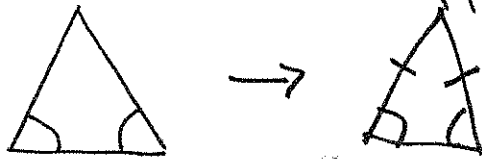
Isosceles Triangle Thm - If two sides of a triangle are congruent, then

the angles opposite those sides are congruent.



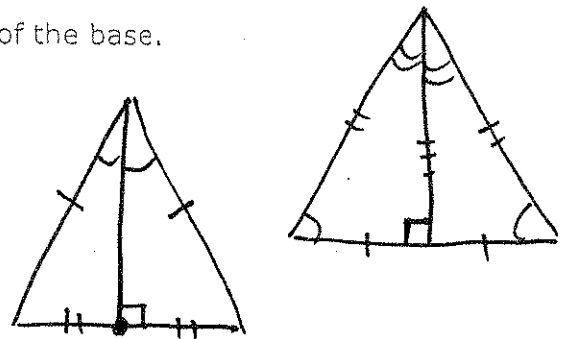
Converse of Isosceles Triangle Thm -

If two angles of a triangle are congruent, then the sides opposite are congruent.



Bisector of Isosceles Triangle Thm - The bisector of the vertex angle of an isosceles triangle

is the perpendicular bisector of the base.



vocab:

Equilateral Triangle

Linear Pair

Complementary angles

Supplementary angles



# 4-6 Skills Practice

## Isosceles Triangles

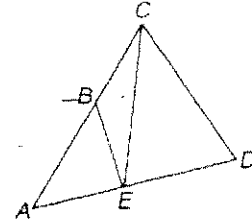
Refer to the figure.

1. If  $\overline{AC} \cong \overline{AD}$ , name two congruent angles.  $\angle D \cong \angle ACD$

2. If  $\overline{BE} \cong \overline{BC}$ , name two congruent angles.  $\angle BCE \cong \angle BEC$

3. If  $\angle EBA \cong \angle EAB$ , name two congruent segments.  $\overline{BE} \cong \overline{AE}$

4. If  $\angle CED \cong \angle CDE$ , name two congruent segments.  $\overline{CE} \cong \overline{CD}$



$\triangle ABF$  is isosceles,  $\triangle CDF$  is equilateral, and  $m\angle AFD = 150$ . Find each measure.

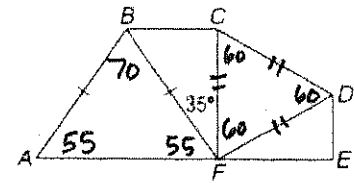
5.  $m\angle CFD$  ~~55~~ 60

6.  $m\angle AFB$  55

7.  $m\angle ABF$  70

8.  $m\angle A$  55

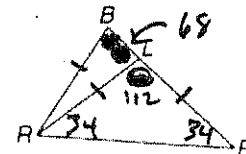
$$\begin{array}{r} 150 \\ - 60 \\ \hline 90 \\ - 35 \\ \hline 55 \end{array}$$



In the figure,  $\overline{PL} \cong \overline{RL}$  and  $\overline{LR} \cong \overline{BR}$ .

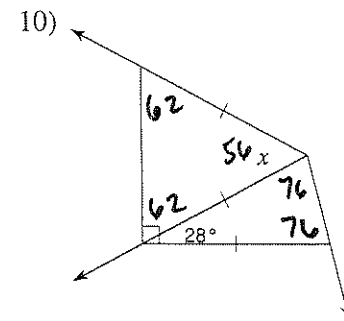
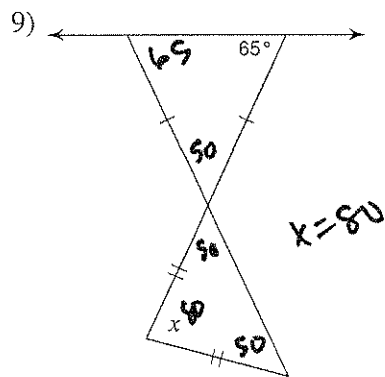
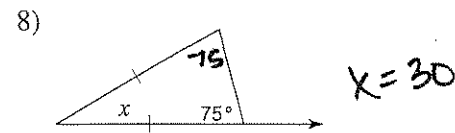
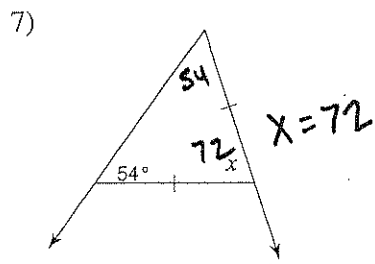
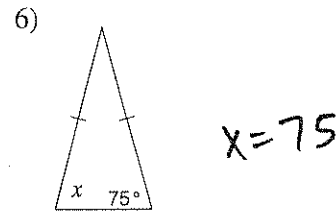
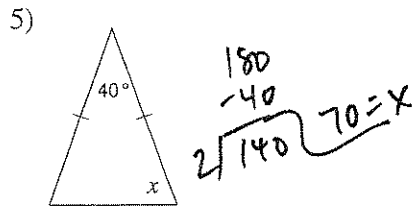
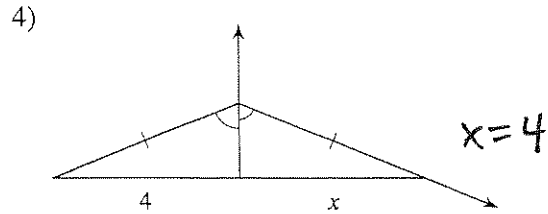
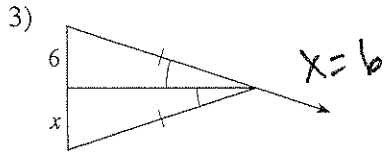
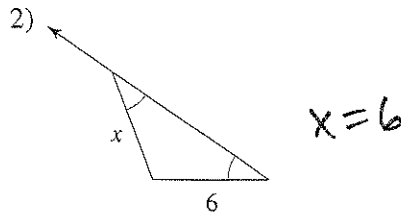
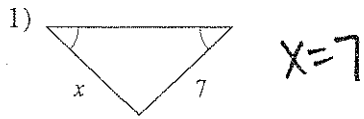
9. If  $m\angle RLP = 100$ , find  $m\angle BRL$ . 20

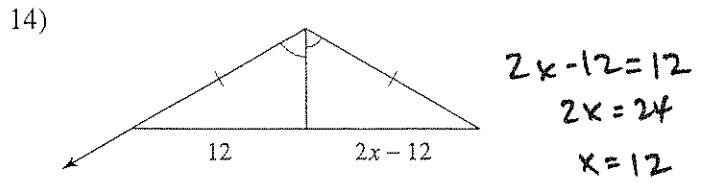
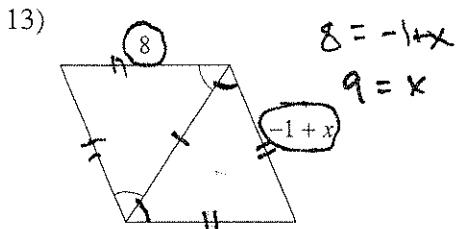
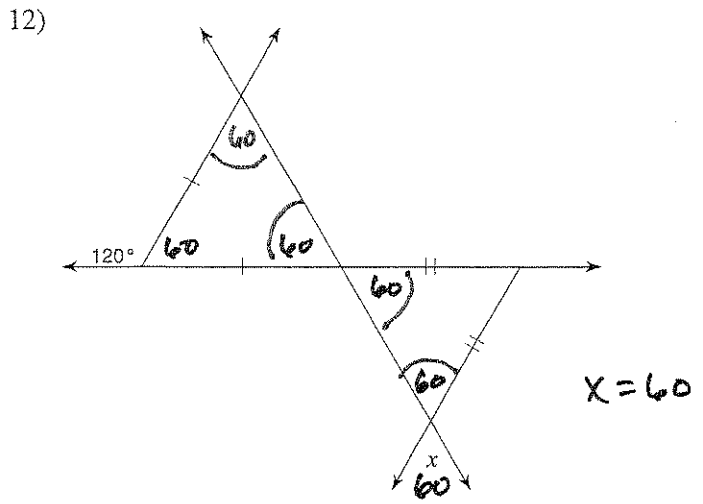
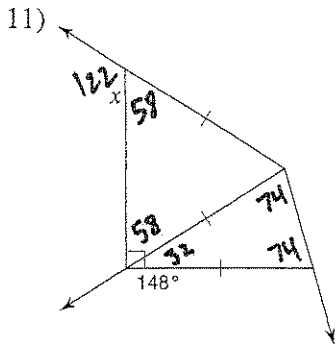
10. If  $m\angle LPR = 34$ , find  $m\angle B$ . 68



Isosceles and Equilateral Triangles

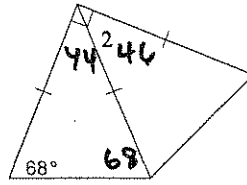
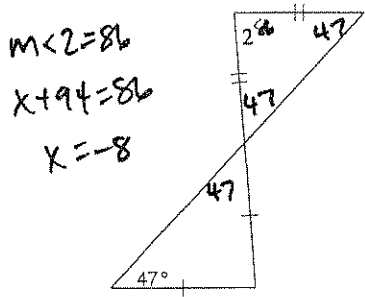
Find the value of  $x$ .





15)  $m\angle 2 = x + 94$

16)  $m\angle 2 = 4x - 2$

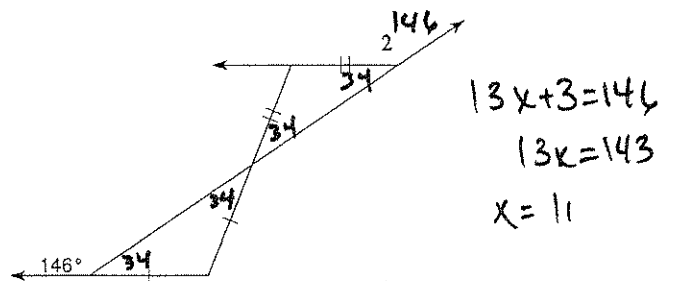
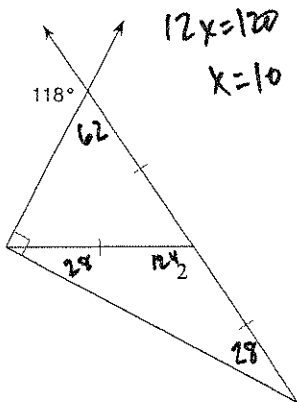


$m\angle 2 = 46$   
 $4x - 2 = 46$   
 $4x = 48$   
 $x = 12$

17)  $m\angle 2 = 12x + 4 = 124$

18)  $m\angle 2 = 13x + 3$

$m\angle 2 = 146$



10

Definition

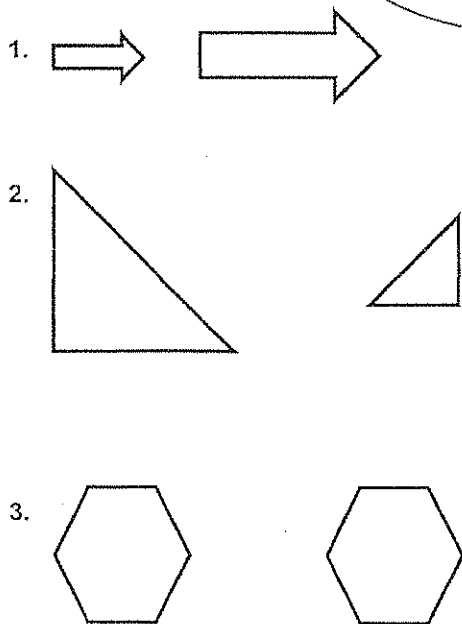
closed figures  
that have  
same shape  
and proportional  
sides

Characteristics

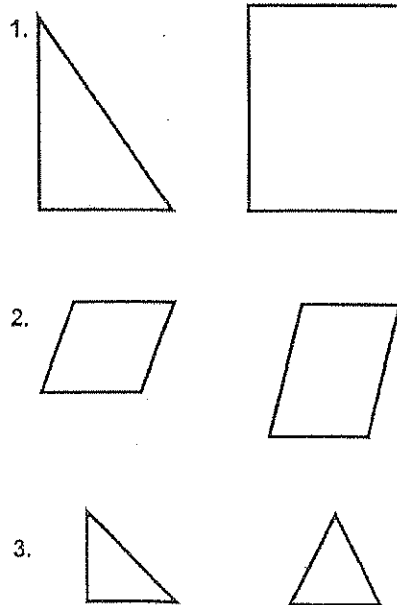
Sides proportional  
angles congruent  
different size (usually)  
same shape (always)

Similar Figures

Examples



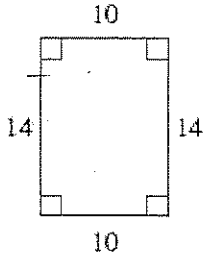
Non-Examples



Similar Polygons

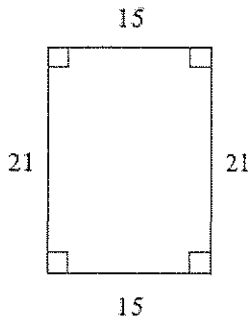
State if the polygons are similar. Explain why or why not for each pair.

1)

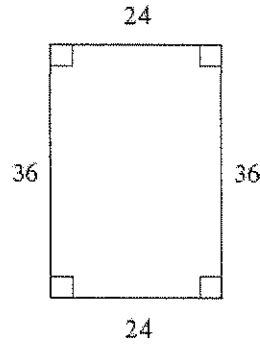


$$\frac{10}{14} = \frac{15}{21}$$

yes

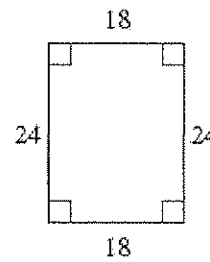


2)



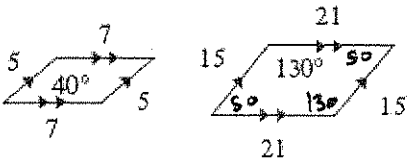
$$\frac{24}{36} \neq \frac{18}{24}$$

no



not proportional sides

3)



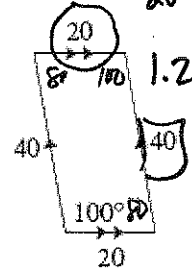
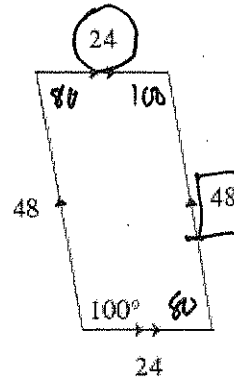
no

angles not equal

4)

$$\frac{24}{20} = \frac{48}{40}$$

yes

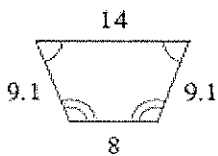


$$\checkmark 24(40) = 20(48)$$

$$\frac{24}{20} = \frac{48}{40}$$

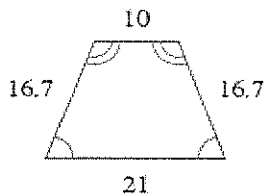
$$1.2 = 1.2 \checkmark$$

5)

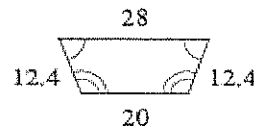


$$\frac{8}{10} \neq \frac{9.1}{16.7} \neq \frac{14}{21}$$

no

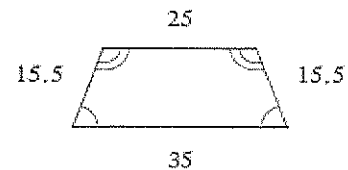


6)

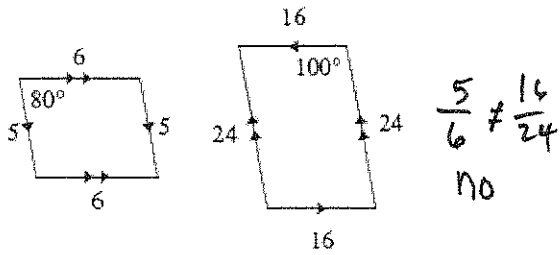


$$\frac{20}{25} = \frac{12.4}{15.5} = \frac{28}{35}$$

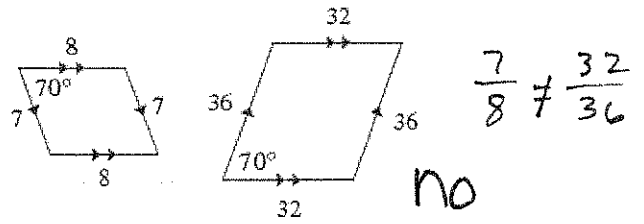
yes



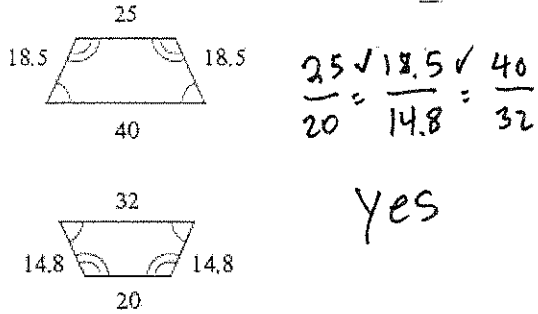
7)



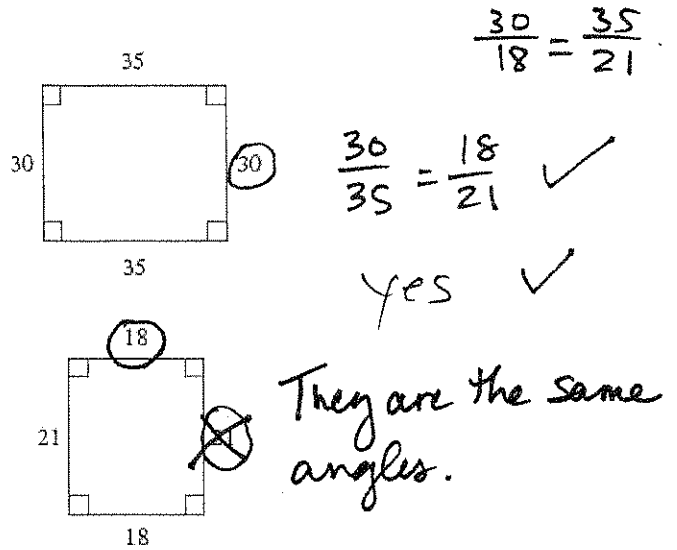
8)



9)

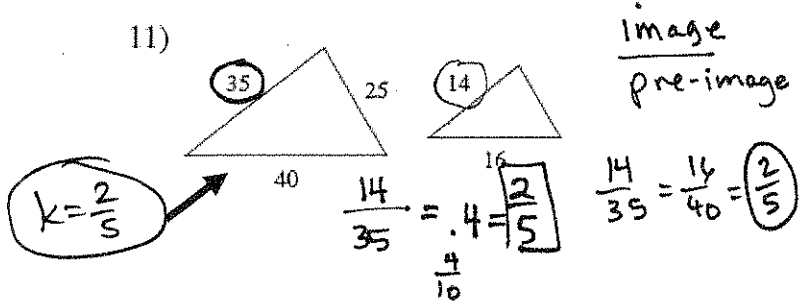


10)

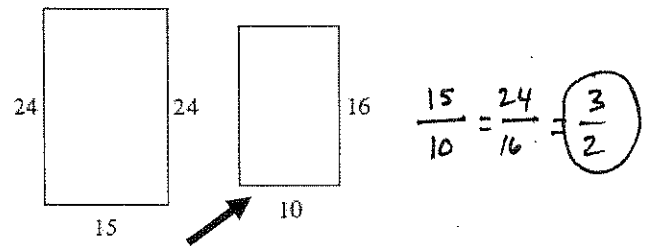


The polygons in each pair are similar. Find the scale factor The pre-image is indicated by an arrow.

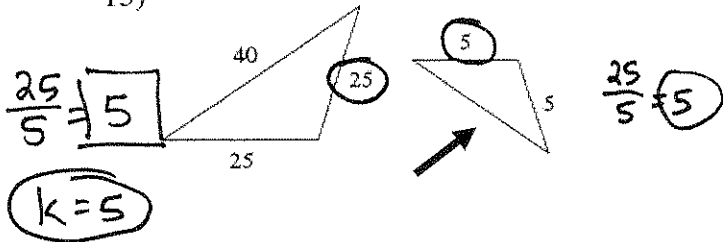
11)



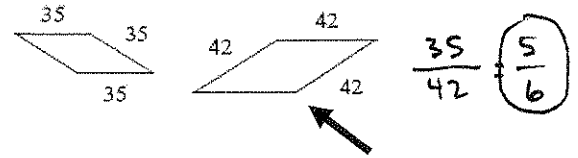
12)



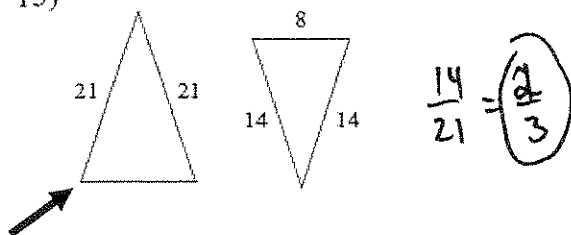
13)



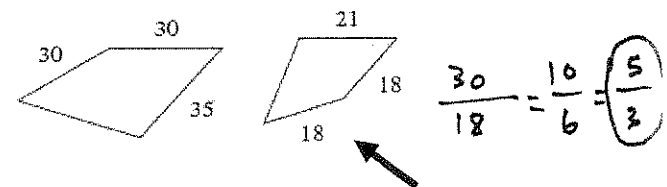
14)



15)

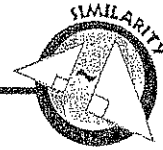


16)



(B)

# 4 FINDING SIDES OF SIMILAR FIGURES

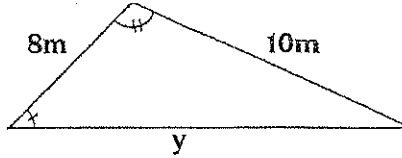
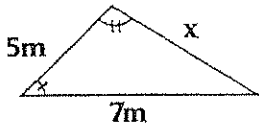


## Structure: RallyCoach

For the following problems, the figures are similar. Find the value of each variable. The figures are not drawn to scale. Round your answers to the nearest tenth.

1.

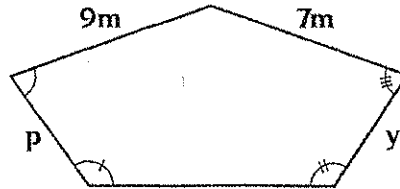
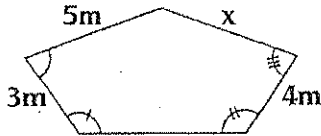
$$\frac{5}{8} = \frac{x}{10}$$



$$x = 6.3$$

$$y = 11.2$$

2.

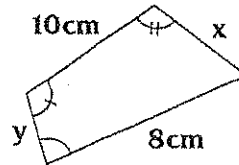
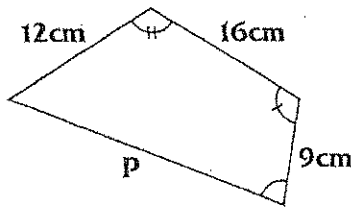


$$x = 3.9$$

$$y = 7.2$$

$$p = 5.4$$

3.

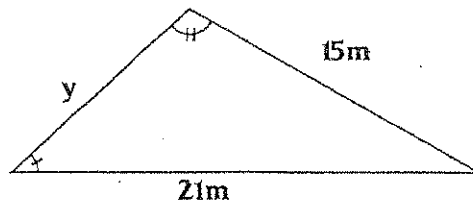
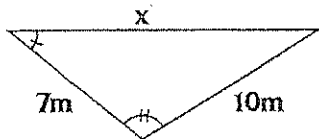


$$x = 7.5$$

$$y = 5.6$$

$$p = 12.8$$

4.



$$x = 14$$

$$y = 10.5$$

14

### 3.3 Warm Up

1. Draw each of the following dilations of quadrilateral BRIA:

- a. 150% scale factor using center X.  $k = 3/2$
- b.  $\frac{3}{2}$  scale factor using center Y.  $k = 3/2$
- c. 1.5 scale factor using center I.
- d. What do you notice?

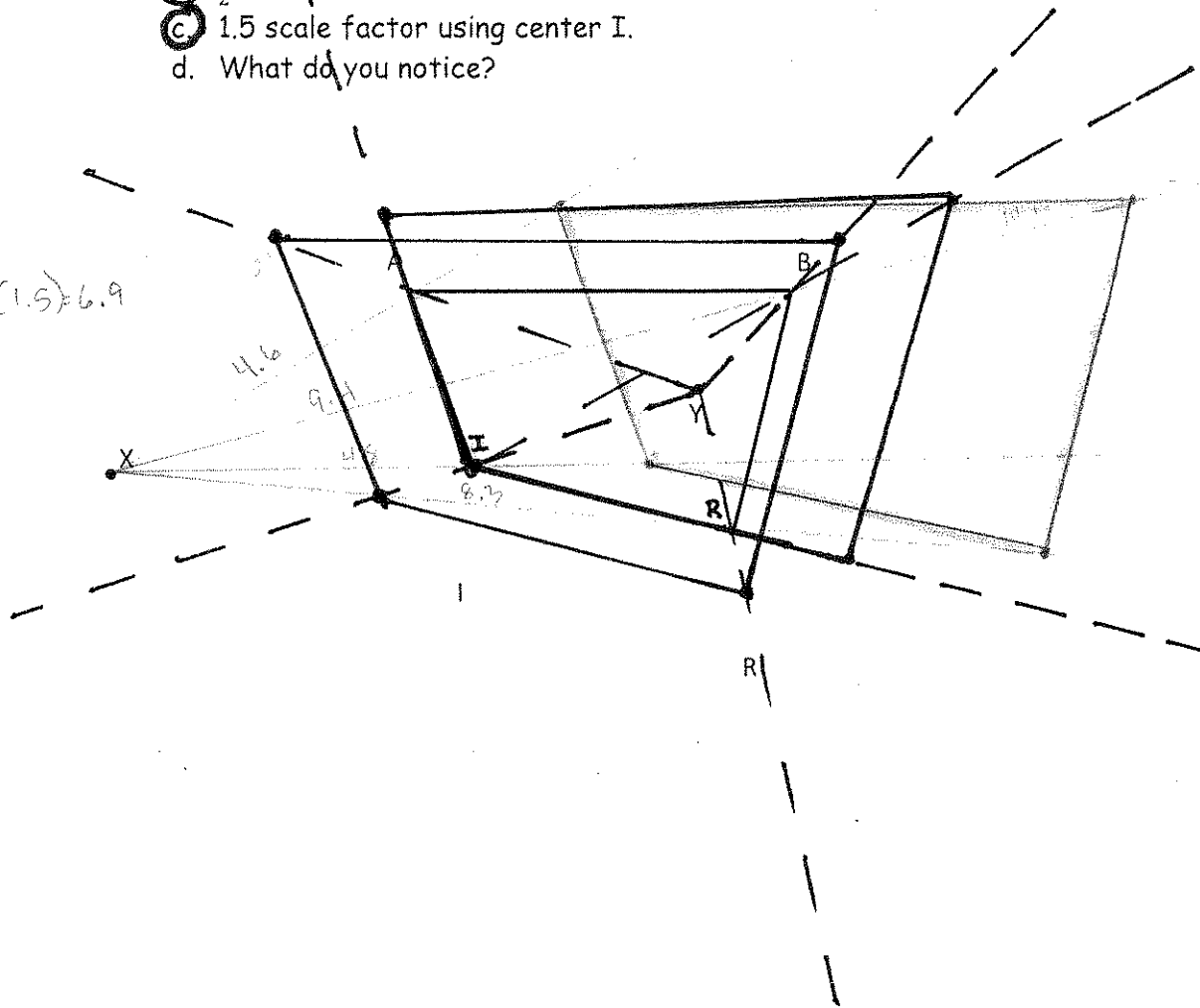
$$3(1.5) = 4.5$$

$$4(1.5) = 6$$

$$1.8(1.5) = 2.7$$

$$1.9(1.5) = 2.85$$

$$4.6(1.5) = 6.9$$



$$3.5(1.5) = 5.25$$

$$4.9(1.5) = 7.35$$

$$2.3(1.5) = 3.45$$

$$(1.5) =$$

- They are all the same size with each other... not with original.
- Larger than original by  $k = 1.5$
- Images are translations of each other

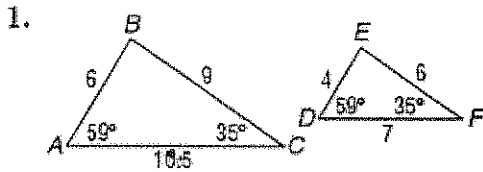
Adapted from *Geometry: A Moving Experience* developed by the Curriculum Research & Development Group, College of Education at the University of Hawaii



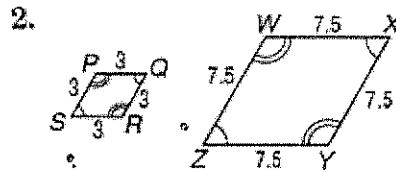
# 7-2 Skills Practice

## Similar Polygons

Determine whether each pair of figures is similar. Justify your answer.

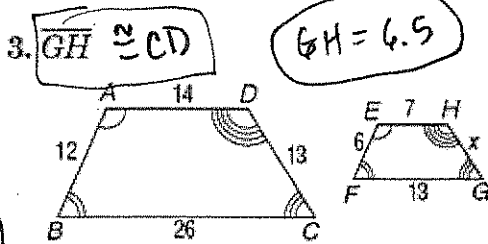


$\frac{6}{4} = \frac{9}{6} = \frac{10.5}{7}$  yes  
 sides proportional  
 angles =



yes  
 sides proportional  
 angles congruent

Each pair of polygons is similar. Write a similarity statement, and find  $x$ , the measure(s) of the indicated side(s), and the scale factor.

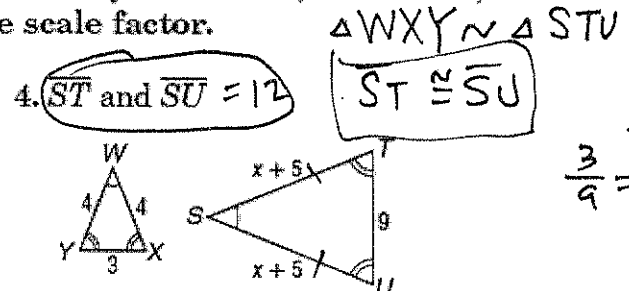


$\frac{14}{7} = \frac{2}{1}$

$\overline{GH} \cong \overline{CD}$

$\overline{GH} = 6.5$

Quad ABCD  $\sim$  Quad EFGH  
 $\frac{13}{2} = x$   
 $6.5 = x$

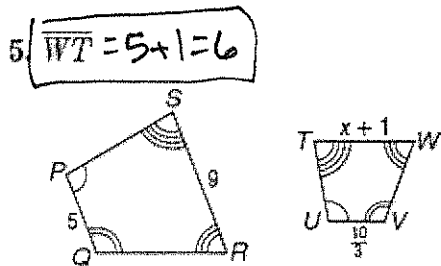


$\overline{ST}$  and  $\overline{SU} = 12$

$\overline{ST} \cong \overline{SU}$

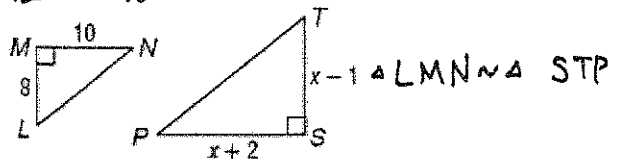
$\frac{3}{9} = \frac{1}{3}$

$\frac{4}{3} = \frac{x+5}{9} = \frac{3(x+5)}{36}$   
 $x+5 = 12$   
 $x = 7$



$\overline{WT} = 5+1 = 6$

$\overline{TS}$  and  $\overline{SP}$



Quad PQRS  $\sim$  Quad UVWT

$5 \div \frac{10}{3}$   
 $5 \cdot \frac{3}{10}$   
 $\frac{15}{10}$   
 $\frac{3}{2}$

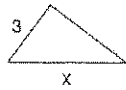
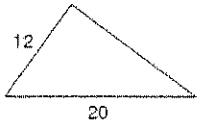
$\frac{5}{9} = \frac{\frac{10}{3}}{x+1}$   
 $5(x+1) = 9(\frac{10}{3})$   
 $5x+5 = 30$   
 $5x = 25$   
 $x = 5$

$\frac{8}{x-1} = \frac{10}{x+2}$   
 $8(x+2) = 10(x-1)$   
 $8x+16 = 10x-10$   
 $26 = 2x$   
 $13 = x$

Similar Figures

Each pair of figures is similar. Find the missing side.

1)

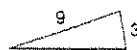


$$\frac{12}{20} = \frac{3}{x}$$

$$12x = 60$$

$$x = 5$$

2)

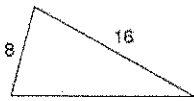
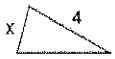


$$\frac{x}{1} = \frac{9}{3}$$

$$3x = 9$$

$$x = 3$$

3)

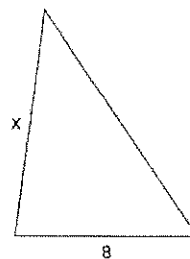
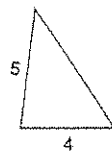


$$\frac{x}{4} = \frac{8}{16}$$

$$16x = 32$$

$$x = 2$$

4)



$$\frac{5}{4} = \frac{x}{8}$$

$$4x = 40$$

$$x = 10$$

5)

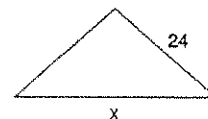
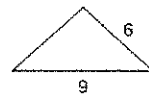


$$\frac{14}{x} = \frac{2}{1}$$

$$2x = 14$$

$$x = 7$$

6)

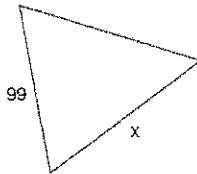
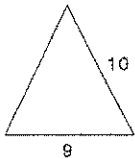


$$\frac{6}{9} = \frac{24}{x}$$

$$6x = 216$$

$$x = 36$$

7)

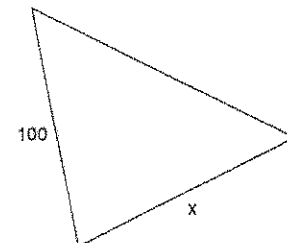
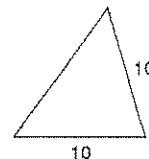


$$\frac{10}{9} = \frac{x}{99}$$

$$9x = 990$$

$$x = 110$$

8)

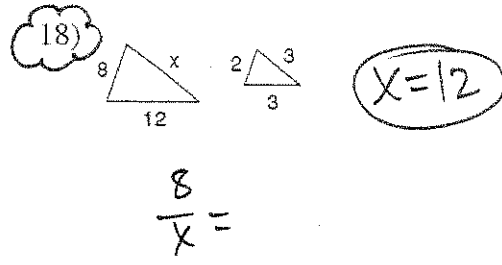
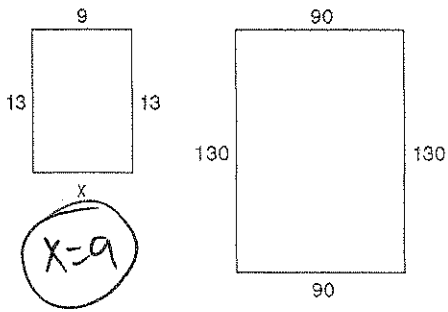


$$\frac{10}{10} = \frac{100}{x}$$

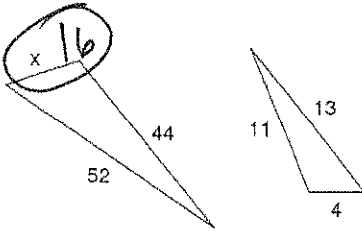
$$x = 100$$

17

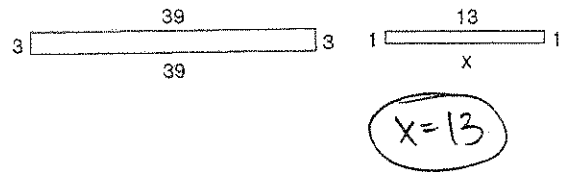
17)



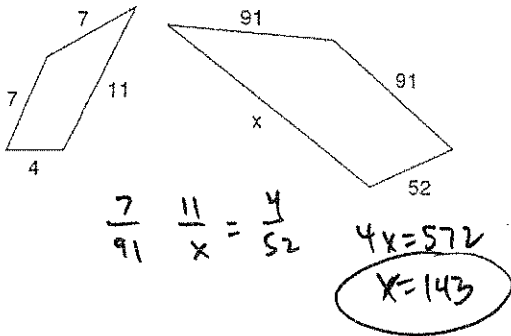
19)



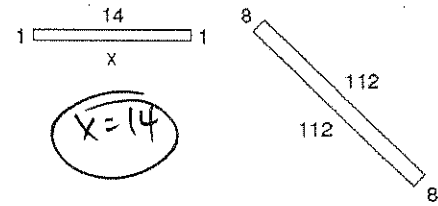
20)



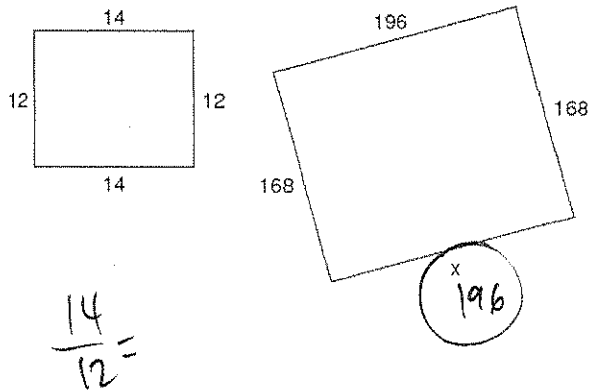
21)



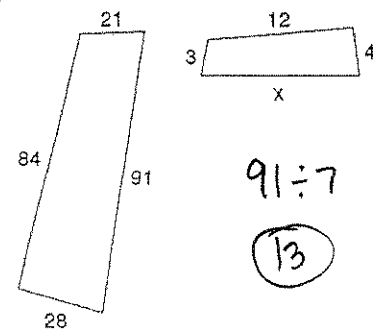
22)



23)



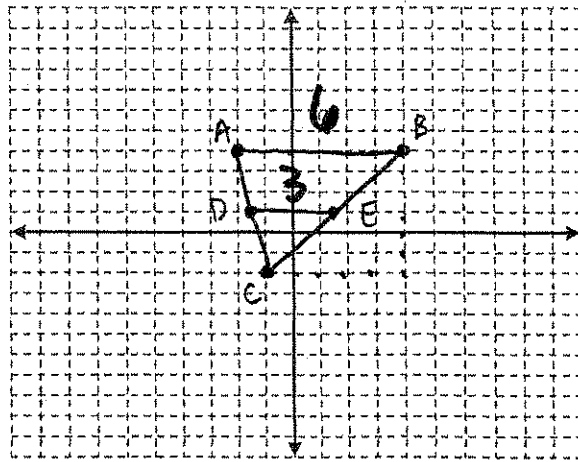
24)



18

Introduction to Midsegments

1. Graph  $A(-2, 4)$ ,  $B(4, 4)$ , and  $C(-1, -2)$ . Draw  $\triangle ABC$ .



down 6  
1 right  
midpoint down 3 right 1/2

2. Determine the midpoints of  $\overline{AC}$  and  $\overline{BC}$ . Add them to the graph of  $\triangle ABC$ . Call these points D and E, respectively, and connect them as a segment.

$$\left( \frac{-2 + -1}{2}, \frac{4 + -2}{2} \right) = \left( \frac{-3}{2}, \frac{2}{2} \right)$$

$E(1.5, 1)$        $D = (-1.5, 1)$

3.  $\overline{DE}$  is referred to as a midsegment. Based on what you did in question 2, why was the prefix "mid" added to "segment"?

The segment that joins the midpoints of two sides of  $\triangle$ .

4. Write a definition of **midsegment**.

5. Calculate the slope of  $\overline{AB}$  and  $\overline{DE}$ . What do these results imply about the two segments?

$\downarrow$        $\downarrow$   
 $m=0$        $m=0$       parallel lines       $\frac{\text{rise}}{\text{run}}$

6. Calculate  $AB$  and  $DE$ . What do these results imply about the two segments?

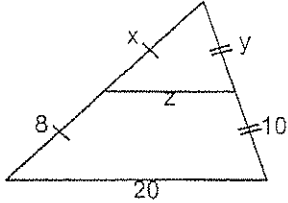
$6$        $3$   
 Midsegment =  $\frac{1}{2}$  Side

7. Write a theorem about midsegments that includes your conclusions from questions 5 and 6.

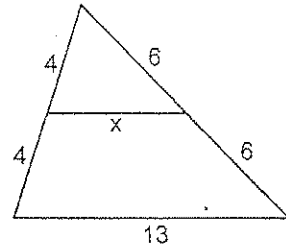
The midsegment is parallel to third side and half of third side.

Directions: Find the values of the variables. You must show all work to receive full credit. Figures are not drawn to scale.

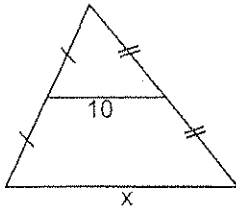
1.  $x = 8$   $y = 10$   $z = 10$



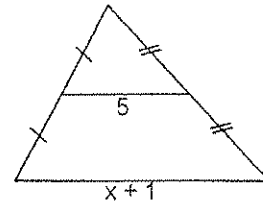
2.  $x = 6.5$



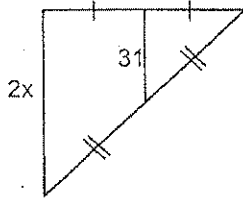
3.  $x = 20$



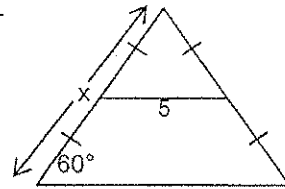
4.  $x = 9$



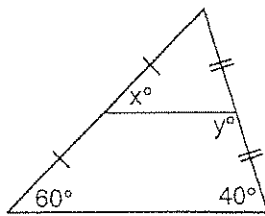
5.  $x = 31$



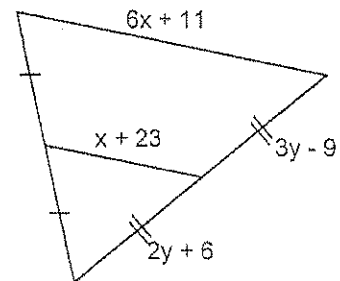
6.  $x = 10$



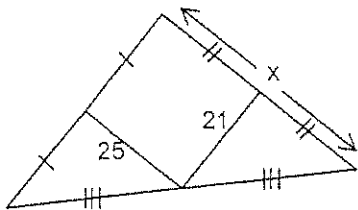
7.  $x = 60$   $y = 140$



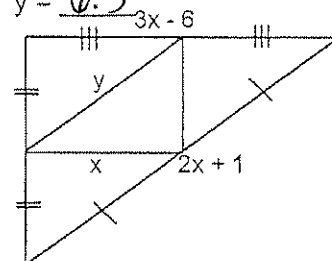
8.  $x = 8.75$   $y = 15$



9.  $x = 50$



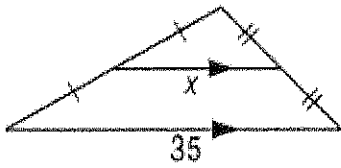
10.  $x = 6$   $y = 6.5$



### 3.5 Midsegment Example Problems

#### Example 1

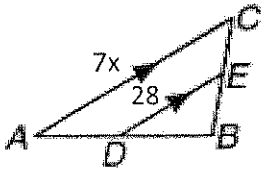
Find  $x$ .



$$x = \frac{35}{2} = 17\frac{1}{2}$$

#### Example 2

DE is the midsegment of  $\triangle ABC$ . Find  $x$ , AC, and ED.



$$7x = 28 \quad (2)$$

$$AC = 7x = 7(8) = 56$$

$$7x = 56$$

$$ED = 28$$

$$x = 8$$

#### Example 3

MN is the midsegment of  $\triangle JKL$ .

$$MN = 2x + 1$$

$$KJ = 5x - 8$$

Find  $x$ , MN, and KJ.

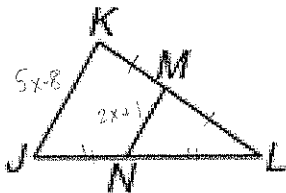
$$2(2x+1) = 5x-8$$

$$MN = 2(10) + 1 = 21$$

$$4x + 2 = 5x - 8$$

$$KJ = 5(10) - 8 = 42$$

$$10 = x$$



#### Example 4

Triangle  $EFG$  has vertices  $E(-4, -1)$ ,  $F(2, 5)$ , and  $G(2, -1)$ . Point  $K$  is the midpoint of  $\overline{EG}$  and  $H$  is the midpoint of  $\overline{FG}$ .

Show that  $\overline{EF}$  is parallel to  $\overline{KH}$ .

$$m = 1 = \frac{6}{6}$$

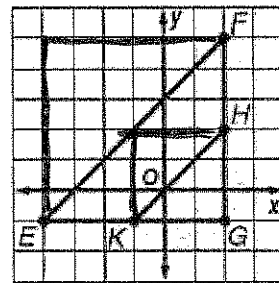
$$m = 1 = \frac{3}{3}$$

The lines are parallel since the slopes are the same.

Show that  $KH = \frac{1}{2}EF$ .

$$3\sqrt{2} = \frac{1}{2}(6\sqrt{2})$$

$$3\sqrt{2} = 3\sqrt{2}$$



(21)

### 3.5 Midsegments - Show What You Know!

1) XY is the midsegment of  $\triangle RST$ . Find each requested measure based on the given information.

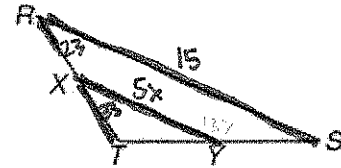
a)  $XY = 16, RS = ?$  **32**

b)  $RS = 22, XY = ?$  **11**

c)  $XY = 5x, RS = 15, x = ?$   **$\frac{3}{2}$**

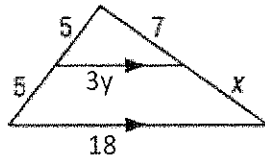
d)  $m\angle R = 23^\circ, m\angle TXY = ?$  **23**

e)  $m\angle XYS = 137^\circ, m\angle YSR = ?$  **43**



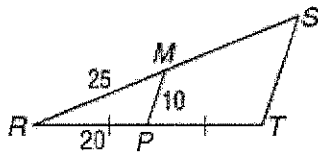
2) Find x and y.

**$x = 7$**



$2(3y) = 18$   
 $6y = 18$   
 $y = 3$

3) Find MS, PT, and ST.



**$MS = 25$**

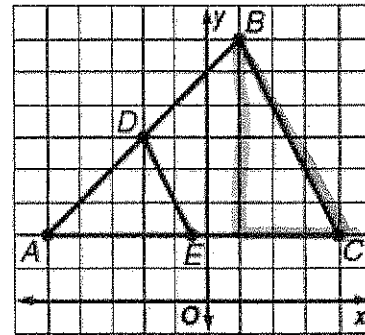
**$PT = 20$**

**$ST = 20$**

4) Triangle ABC has vertices  $A(-5, 2)$ ,  $B(1, 8)$ , and  $C(4, 2)$ . Point D is the midpoint of  $\overline{AB}$  and E is the midpoint of  $\overline{AC}$ .

a) Identify the coordinates of D and E.

**$(-2, 5)$   $(-\frac{1}{2}, 2)$**



b) Show that  $\overline{BC}$  is parallel to  $\overline{DE}$ .

$m_{BC} = \frac{1}{3} = -2$   
 $m_{DE} = \frac{3}{1.5} = -2$   
} same slope means they are parallel

c) Show that  $DE = \frac{1}{2}BC$ .

$3.35 = \frac{1}{2}(6.7)$   
 $\checkmark 3.35 = 3.35$

$DE^2 = 3^2 + 1.5^2$   
 $DE^2 = 9 + 2.25$   
 $\sqrt{DE^2} = \sqrt{11.25}$   
 $DE = 3.35$

$BC^2 = 6^2 + 3^2$   
 $BC^2 = 36 + 9$   
 $BC^2 = 45$   
 $BC = \sqrt{45}$   
 $= 6.71$

$3.35(2) = 6.7 \checkmark$

**22**

# Homework

## I. Short Answer

1. What are the properties of a midsegment of a triangle?

The midsegment is parallel to the third side and half of the third side.

2. The midsegment of a triangle has endpoints  $(-2,6)$  and  $(4,12)$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- (a) What is the slope of the base of the triangle?

$$m = \frac{12 - 6}{4 - (-2)} = \frac{6}{6} = 1$$

- (b) What is the approximate length of the base?

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

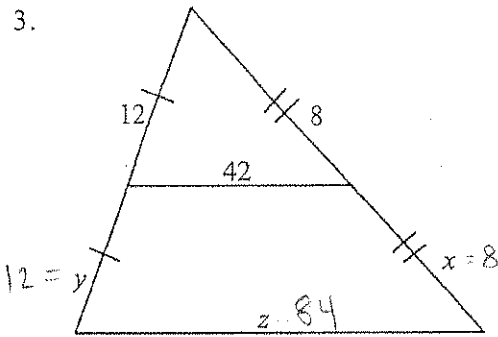
$$d = \sqrt{(-2 - 4)^2 + (6 - 12)^2}$$

$$d = \sqrt{36 + 36}$$

$$d = \sqrt{72} \approx 8.5$$

## II. Solve for the missing variables:

3.

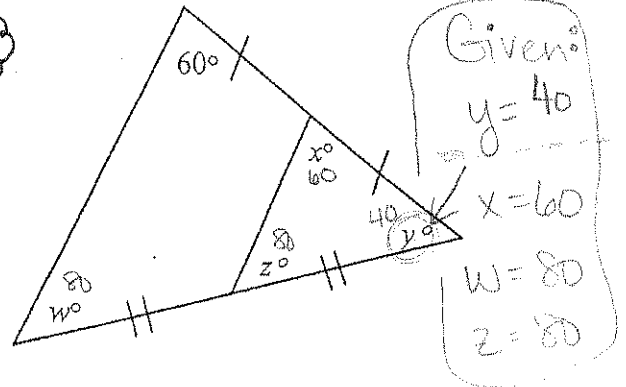


$$x = 8$$

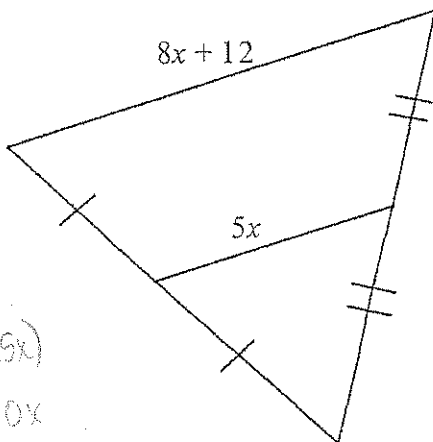
$$y = 12$$

$$z = 84$$

4.



5.



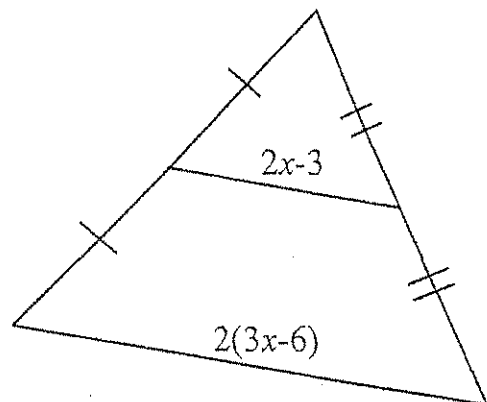
$$8x + 12 = 2(5x)$$

$$8x + 12 = 10x$$

$$12 = 2x$$

$$6 = x$$

6.



$$3x - 6 = 2x - 3$$

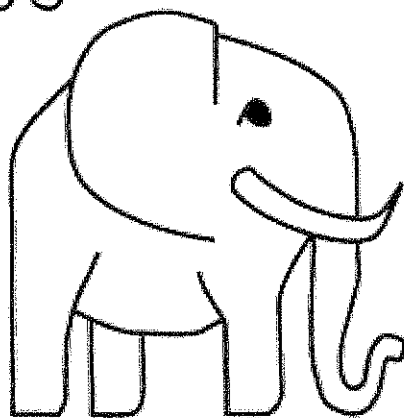
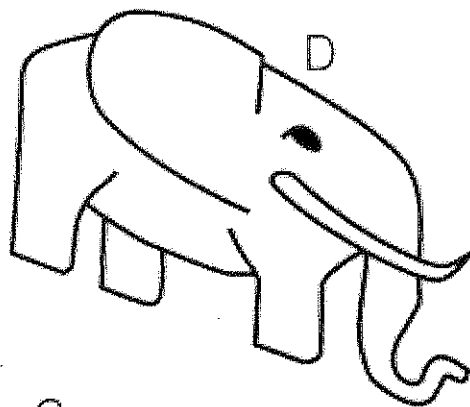
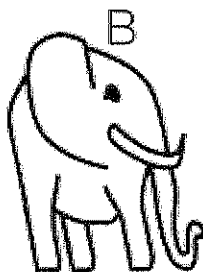
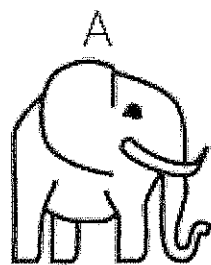
$$x = 3$$

(23)



# Comparing Elephants

Compare each elephant with elephant A. Describe the differences and similarities between elephant A and each of the other elephants.



Compare with elephant A:

(B) identical

(C) reflection  
y axis

(D) stretched  
horizontal

(E) dilation

(F) stretched +  
compressed

(G) equal, same  
dilation  $k=1$   
congruent

(H) stretched  
vertically

(I) dilation

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Congruent Triangles and Rigid Motion

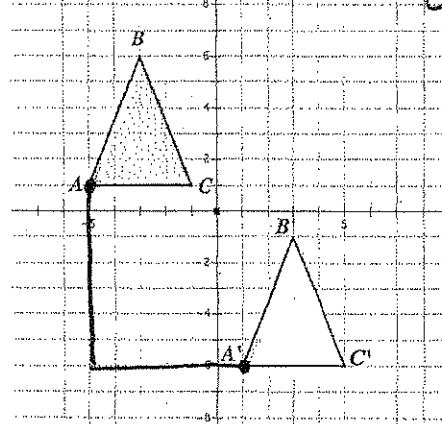
Name \_\_\_\_\_

Directions: The following problems deal with congruency and rigid motion. The term "rigid motion" is also known as "isometry" or "congruence transformations."

1. In the diagram at the right, a transformation has occurred on  $\triangle ABC$ .  
 a) Describe a transformation that created image  $\triangle A'B'C'$  from  $\triangle ABC$ .

translation  
 down 7 + right 6

$(x+6, y-7)$

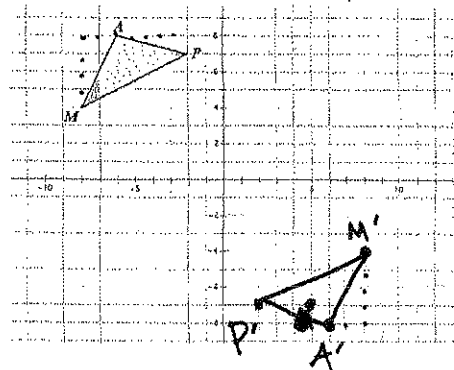


- b) Is  $\triangle ABC$  congruent to  $\triangle A'B'C'$ ? yes Explain.

Same size  
 Same shape

2. The vertices of  $\triangle MAP$  are  $M(-8, 4)$ ,  $A(-6, 8)$  and  $P(-2, 7)$ .  
 The vertices of  $\triangle M'A'P'$  are  $M'(8, -4)$ ,  $A'(6, -8)$  and  $P'(2, -7)$ .

- a) Plot  $\triangle M'A'P'$ .



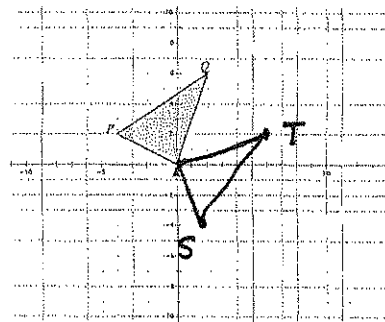
- b) Verify that the triangles are congruent.

- c) Describe a rigid motion that can be used to  $M'A'P'$

rotation

3. Given  $\triangle PQR$  with  $P(-4, 2)$ ,  $Q(2, 6)$  and  $R(0, 0)$  is congruent to  $\triangle STR$  with  $S(2, -4)$ ,  $T(6, 2)$  and  $R(0, 0)$ .

- a) Plot  $\triangle STR$ .



- b) Describe a rigid motion which can be used to verify the triangles are congruent.

reflection

Definition

closed figures  
that have equal  
sides and angles

Characteristics

Same  
Same size  
Same shape  
Same angles / degrees  
Same lengths  
dilation with  $k=1$   
reflection rotation  
translation

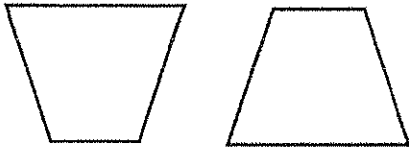
Congruent  
Figures

Examples

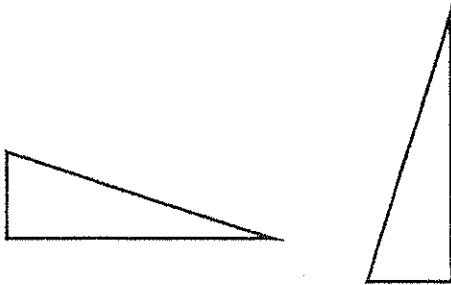
1.



2.



3.

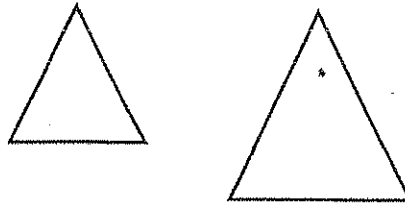


Non-Examples

1.

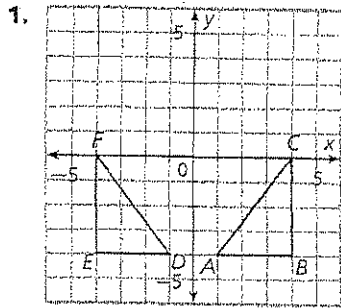


2.

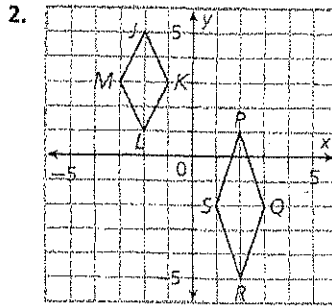


# PRACTICE

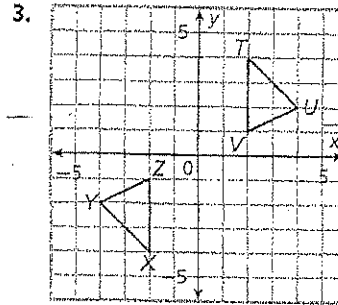
Use the definition of congruence in terms of rigid motions to determine whether the two figures are congruent and explain your answer.



yes  
reflected y-axis

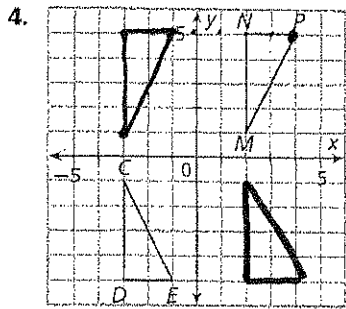


no

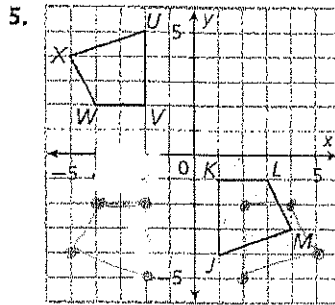


yes  
rotation 180°

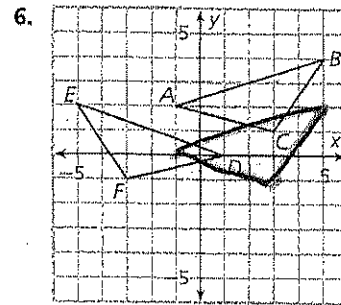
For each pair of congruent figures, find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.



reflect x axis  
translate right 5  
OR  
reflect x axis  
translate left 5



rotation 180°  
reflect x axis  
reflect y axis  
translate up 1  
left 1



reflect y axis  
translate up 2

7.  $\triangle ABC \cong \triangle DEF$  and  $\triangle DEF \cong \triangle GHJ$ . Can you conclude  $\triangle ABC \cong \triangle GHJ$ ? Explain.

yes Substitution Property  
Transitive Property

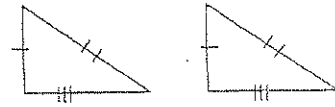
$$\triangle ABC \cong \triangle DEF \cong \triangle GHJ$$



Geometry Notes

4.2 & 4.3 Proving Triangles Congruent

Side-Side-Side (SSS) Congruence Postulate



Are the following congruent?

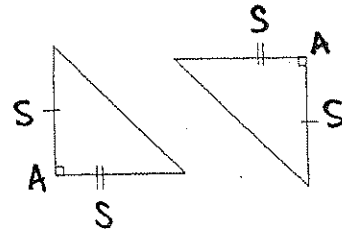
yes, by SSS



Common side  
reflexive property

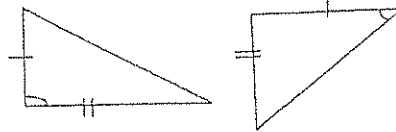
Side-Angle-Side (SAS) Congruence Postulate

angle between  
the two sides



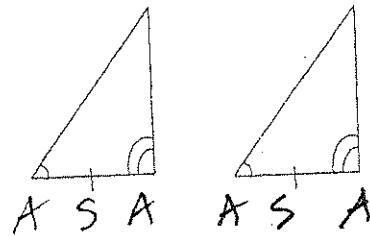
Are the following congruent?

no



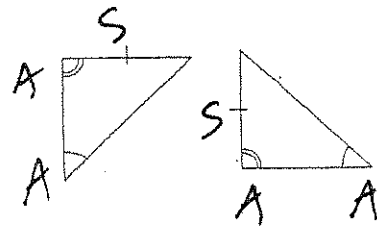
Angle-Side-Angle (ASA) Congruence Postulate

side between the  
two angles

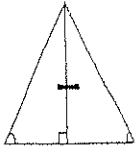


Angle-Angle-Side (AAS) Congruence Postulate

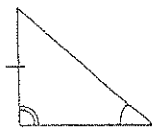
side not between  
two angles



Are the following sets of triangles congruent?



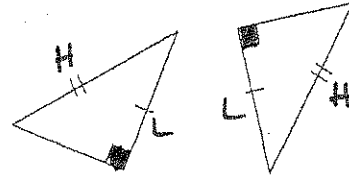
yes  
AAS



no

AAS  $\neq$  ASA

Hypotenuse Leg (HL) Congruence Postulate



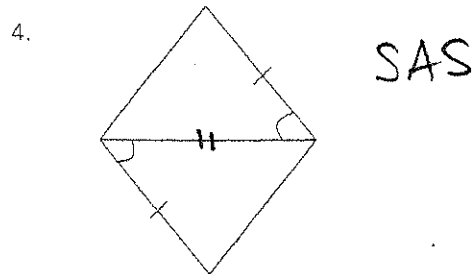
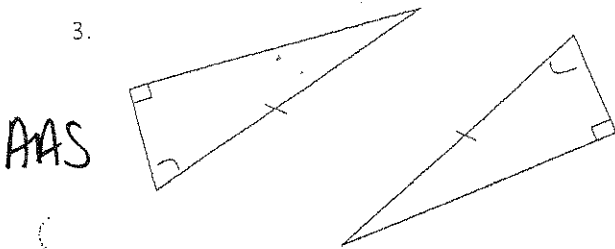
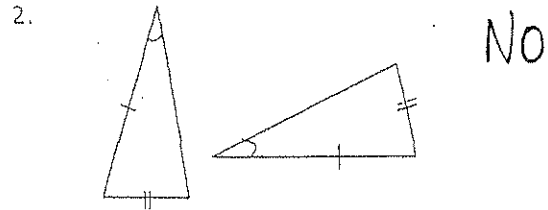
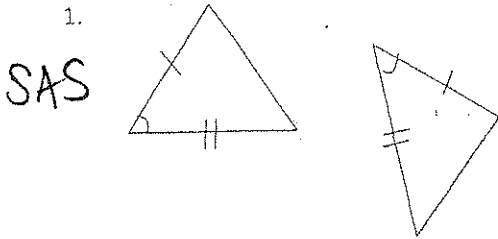
One type that doesn't work.

ASS, SSA

The second type that doesn't work.

AAA

Examples: Tell whether the triangles are congruent. If so, state the theorem or postulate used to prove it.

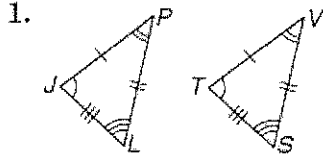


# 4-3 Skills Practice

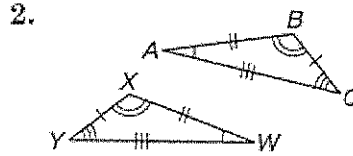
## Congruent Triangles

CPTC - Corresponding Parts of Congruent Triangles are Congruent.

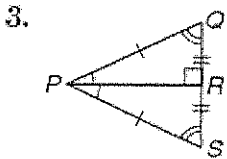
Identify the congruent triangles in each figure.



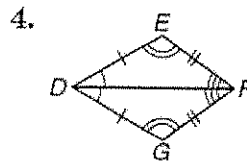
$\triangle JLP \cong \triangle TSV$



$\triangle ABC \cong \triangle WXY$



$\triangle PQR \cong \triangle PSR$



$\triangle DEF \cong \triangle DGF$

Name the congruent angles and sides for each pair of congruent triangles.

5.  $\triangle ABC \cong \triangle FGH$

$\angle A \cong \angle F$   
 $\angle B \cong \angle G$   
 $\angle C \cong \angle H$

$\overline{AB} \cong \overline{FG}$   
 $\overline{BC} \cong \overline{GH}$   
 $\overline{AC} \cong \overline{FH}$

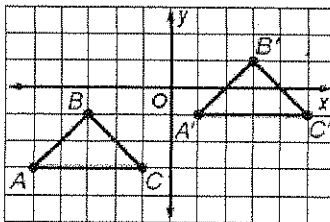
6.  $\triangle PQR \cong \triangle STU$

$\angle P \cong \angle S$   
 $\angle Q \cong \angle T$   
 $\angle R \cong \angle U$

$\overline{PQ} \cong \overline{ST}$   
 $\overline{QR} \cong \overline{TU}$   
 $\overline{PR} \cong \overline{SU}$

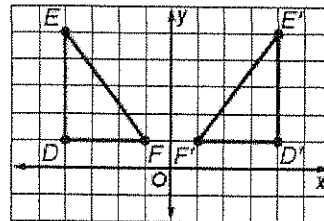
Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7.  $\triangle ABC \cong \triangle A'B'C'$



translation  
 6 right + up 2  
 $(x+6, y+2)$

8.  $\triangle DEF \cong \triangle D'E'F'$

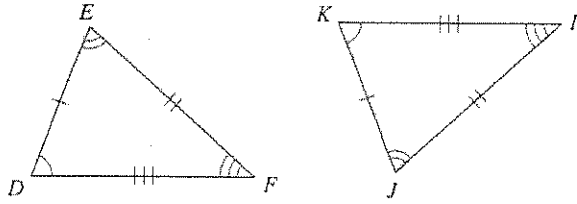


reflection  
 y axis

Congruence and Triangles

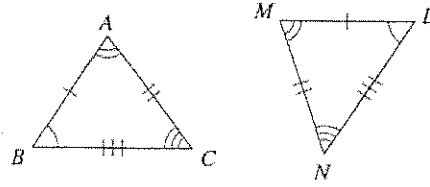
Complete each congruence statement by naming the corresponding angle or side.

1)  $\triangle DEF \cong \triangle KJI$



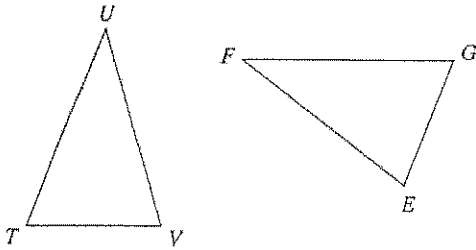
$\overline{FD} \cong ? \overline{JK}$

2)  $\triangle BAC \cong \triangle LMN$



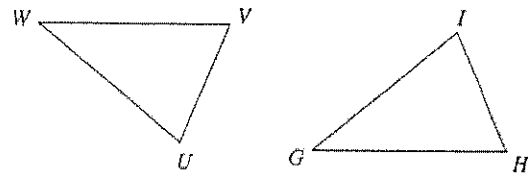
$\angle A \cong ? \angle M$

3)  $\triangle TUV \cong \triangle GFE$



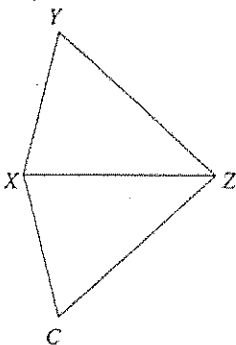
$\angle U \cong ? \angle F$

4)  $\triangle WVU \cong \triangle GHI$



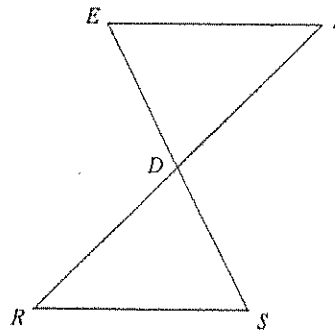
$\angle W \cong ? \angle G$

5)  $\triangle ZXY \cong \triangle ZXC$



$\angle Y \cong ? \angle C$

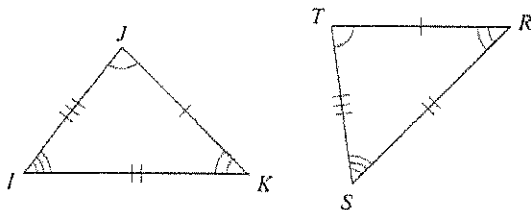
6)  $\triangle DEF \cong \triangle DSR$



$\angle F \cong ? \angle R$

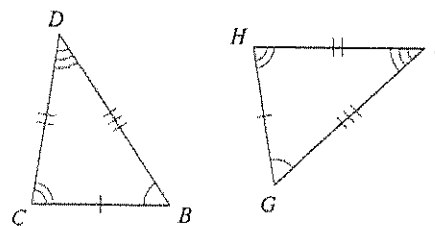
Write a statement that indicates that the triangles in each pair are congruent.

7)



$\triangle IJK \cong \triangle \underline{STR}$

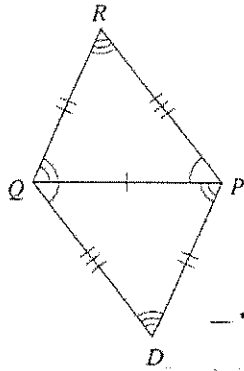
8)



$\triangle BCD \cong \triangle \underline{GHI}$

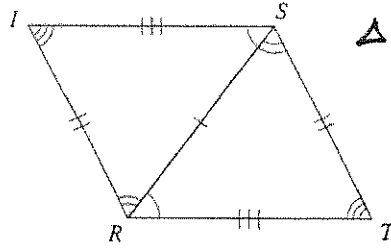


9)



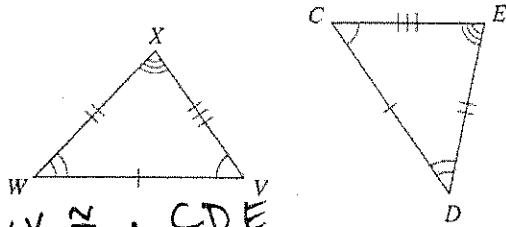
$\triangle PQR \cong \triangle QPD$   
 $\triangle QPR \cong \triangle PQR$   
 $\triangle RQP \cong \triangle DPQ$   
 $\triangle QRP \cong \triangle PDR$

10)



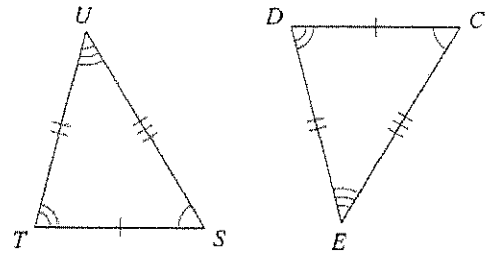
$\triangle IRS \cong \triangle TSR$

11)



$\triangle VWX \cong \triangle CDE$

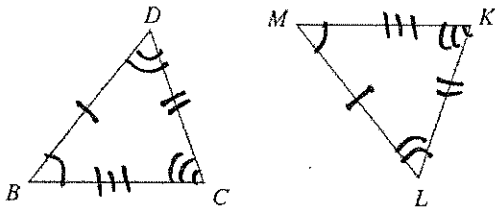
12)



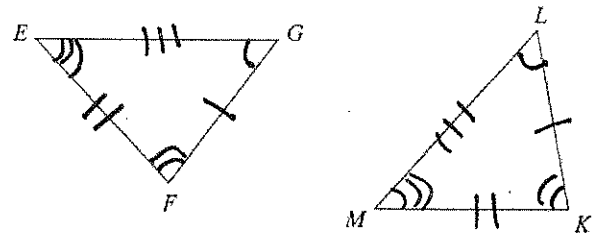
$\triangle UTS \cong \triangle CDE$

Mark the angles and sides of each pair of triangles to indicate that they are congruent.

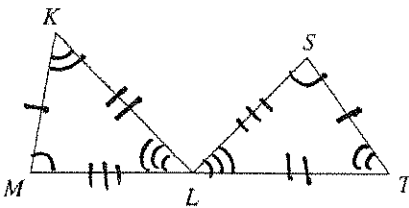
13)  $\triangle BDC \cong \triangle MLK$



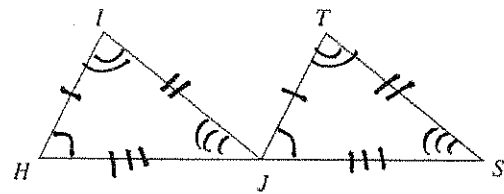
14)  $\triangle GFE \cong \triangle LKM$



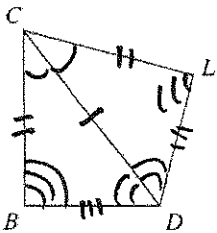
15)  $\triangle MKL \cong \triangle STL$



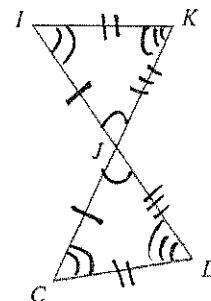
16)  $\triangle HIJ \cong \triangle JTS$



17)  $\triangle CDB \cong \triangle CDL$



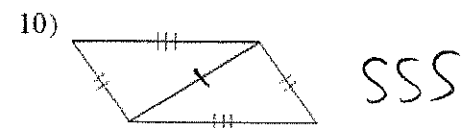
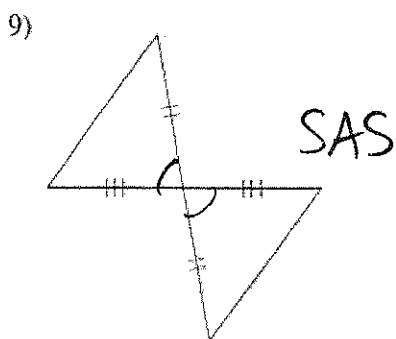
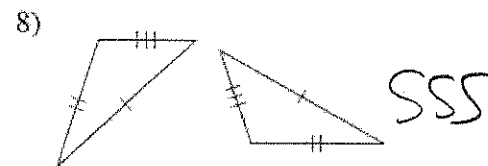
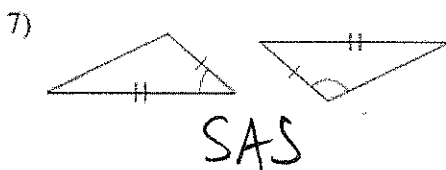
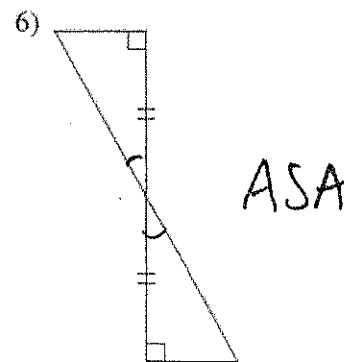
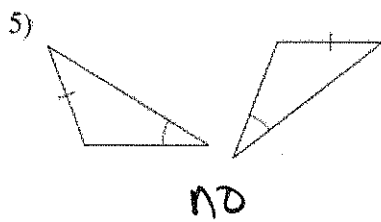
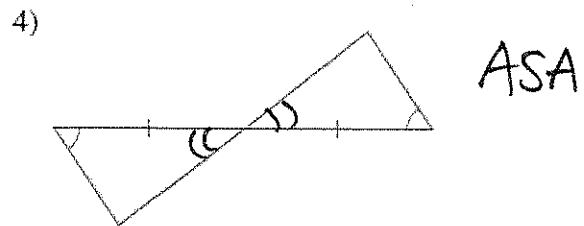
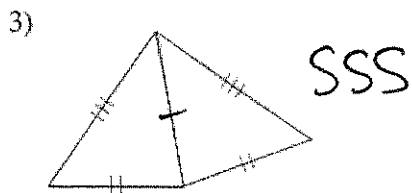
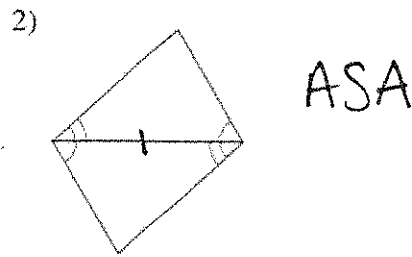
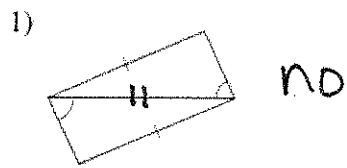
18)  $\triangle JIK \cong \triangle JCD$



32

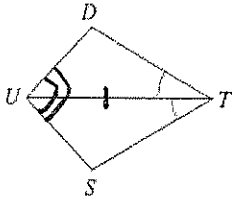
### SSS, SAS, ASA, and AAS Congruence

State if the two triangles are congruent. If they are, state how you know.

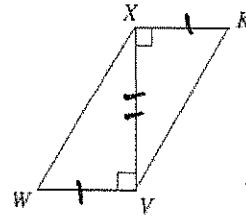


State what additional information is required in order to know that the triangles are congruent for the reason given.

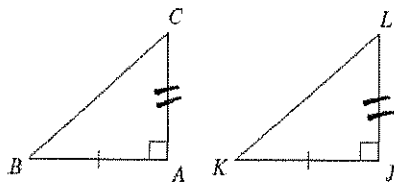
11) ASA



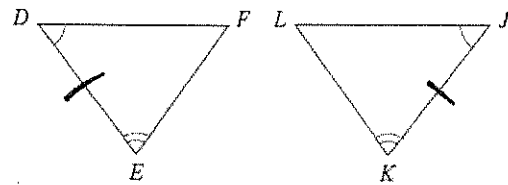
12) SAS



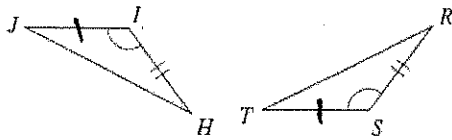
13) SAS



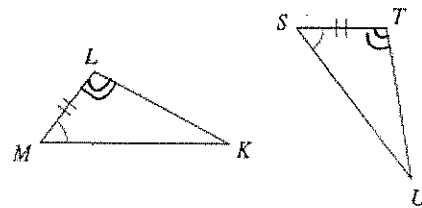
14) ASA



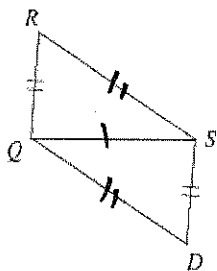
15) SAS



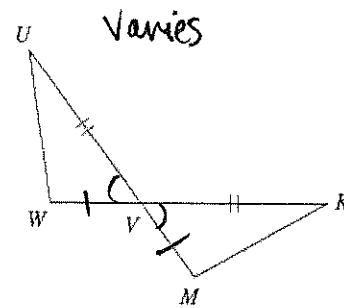
16) ASA

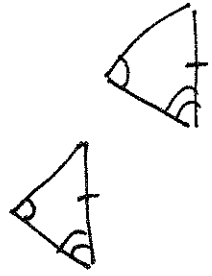
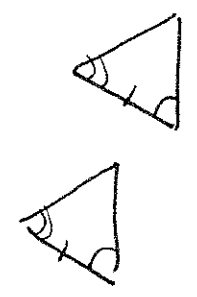
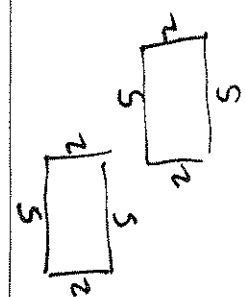
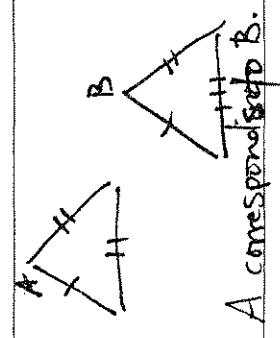


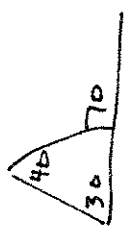
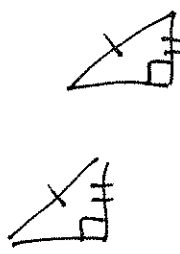
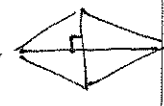
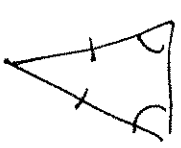
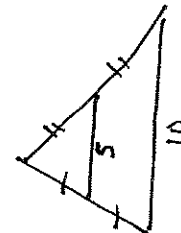
17) SSS



18) SAS



Vocabulary Word	Definition	Characteristics	Picture and/or Symbol	Real Life Examples
Angle-Angle-Side AAS	Two angles and unincluded side of one triangle is congruent to two angles and unincluded side of another triangle.	Side is not between the angles		Architecture
Angle-Side-Angle ASA	Two angles and the side between them are congruent in two triangles.	The congruent side is between the angles.		Modeling
congruent	All sides and angles of a figure are equal.	equal Same size & shape		Manufacturing Business
CPCTC	Corresponding parts of congruent triangles are congruent.	All sides equal. All angles equal.	 A corresponds to B.	Matching, Equivalence

Vocabulary Word	Definition	Characteristics	Picture and/or Symbol	Real Life Examples
Exterior Angles Theorem	The exterior angle is equal to sum of remote interior angles.	1 exterior angle equals sum of 2 interior angles		Construction
Hypotenuse Leg HL	Two triangles have $\cong$ hypotenuses and set of legs.	It must be right triangle.		Kites 
Isosceles Triangle Theorem	If two sides of $\Delta$ are $\cong$ , then the base angles are congruent.	Two $\cong$ sides Two $\cong$ angles		Pitch of roof Art
Mid-segment	The segment that joins two midpoints in a triangle.	It is parallel + half of the base.		Engineering

Vocabulary Word	Definition	Characteristics	Picture and/or Symbol	Real Life Examples
proportion	An equation with 2 ratios	$\frac{2}{5} = \frac{6}{15}$ $30 = 30$		Dilations Clipart Logos
Side-Angle-Side SAS	When 2 sets of sides are congruent and the angle between them are equal in two triangles.	Angle must be between the two sides.		Bridges Art Construction
scale factor	The ratio of two similar figures.	Multiply or compare corresponding sides		Art Design Models
similar	Figures with same shape but different size. (dilation)	dilations sides proportional angles congruent		Pyramids Land sections

Vocabulary Word	Definition	Characteristics	Picture and/or Symbol	Real Life Examples
Side-Side-Side SSS	3 sides of 1 $\Delta$ are equal to 3 sides in another $\Delta$	All sides are the same (SSSS)		Mass Production
Triangle Angle Sum Theorem	Sum of the interior angles is equal to 180 degrees	Three angles add to 180. $\underline{\quad} + \underline{\quad} + \underline{\quad} = 180$		Construction Design
Isosceles Triangle	A triangle with at least two equal sides.	Two angles are equal.		Pennants
Equilateral Triangle	A triangle with 3 congruent sides & angles.	All angles are 60°.		Doritos Roof